2D-MESH GENERATORS

CM2 TRIAMESH & CM2 QUADMESH

CM2 TRIAMESH ANISO & CM2 QUADMESH ANISO

Series 4.6.x

TUTORIAL

AND

USER'S MANUAL

Computing Objects

25 rue du Maréchal Foch, F-78000 Versailles.
www.computing-objects.com

Revision of manual: October 2015

Figures

Tables

This manual describes the 2-D mesh generators of the **CM2 MeshTools** library: the isotropic meshers **CM2 TriaMesh** and **CM2 QuadMesh**, and the anisotropic meshers **CM2 TriaMesh Aniso** and **CM2 QuadMesh Aniso**. Cases of 3-D surface mesh using the anisotropic meshers are also presented.

All these mesh generators are *constrained unstructured* meshers: the boundary mesh (contour mesh) as well as the internal hard edges and hard points (embedded) are kept unmodified in the final mesh.
Based on a fast and robust hybrid "Advancing-Front and Delaunay" algorithm, they generate high quality elements with smooth grading size according to the length of the boundary edges or to the user-specified sizes. The speed is near independent of the number of the elements to be generated.

Option switches can be used to adapt the meshers to the various needs of the user concerning mesh generation, refinement and optimization (they can also be used as optimizers-only of some already existing meshes).
The quad meshers can generate *all-quad* meshes (the default) or mixed *quad-dominant* meshes. In the all-quad mode, the number of edges in each segment must be even. This condition can be easily respected using the functions of CM2 Meshtools1D. However, when parity is not suitable or even impossible to achieve, the quad-dominant mode must be activated and user must accept some triangles in the final mesh.

Many data concerning the mesh are available upon exit: shape and size qualities histograms, matrix of the neighbors, number of subdomains, meshed surface...

Like all the other meshers of the library, CM2 TriaMesh, CM2 QuadMesh, CM2 TriaMesh Aniso and CM2 QuadMesh Aniso are thread-safe (several instances or the meshers can be ran concurrently).

The `math1` library exports the vector and matrix classes needed to communicate with the meshers.
The additional libraries `meshtools`, `meshtools1d`, `meshtools2d` and `meshtools3D` can be used to generate simple 1-D meshes for the boundaries or to do some mesh transformations (translation, rotations, concatenation, merging...)

For maximum performance, these software components are developed using the standard C++ language with efficient object-oriented programming techniques.

The full sources are available and they have been ported to most major platforms.
With a binary license, these libraries are shipped as precompiled dynamic libraries - DLL Win32 or shared Linux i386 - with .lib and C++ headers files.

The source code of CM2 MeshTools (full library) has been registered with the APP under Inter Deposit number IDDN.FR.001.260002.00.R.P.1998.000.20700 (22/06/1998) and
IDDN.FR.001.250030.00.S.P.1999.000.20700 (16/06/1999) is regularly deposited since then.

The source code specific to CM2 TriaMesh, together with this manual, has been registered with the APP under Inter Deposit number IDDN.FR.001.440021.000.R.P.2008.000.20700 (31/10/2008) and is regularly deposited since then.

The source code specific to CM2 QuadMesh, together with this manual, has been registered with the APP under Inter Deposit number IDDN.FR.001.440020.000.R.P.2008.000.20700 (31/10/2008) and is regularly deposited since then.

I – OVERVIEW OF THE MATH1 LIBRARY

Before digging into the meshers, let us have a look into the `math1` library which exports the types of vector and matrix used by the meshers. We do not intend here to give a full description of this `math1` library, nor the associated template libraries `vecscal`, `vecvec`, `matscal`, `matvec`, and `matmat`. We will simply explain the basic traits of these math classes in order to use the meshers properly. A full description can be found in the math1 API[1].

The `math1` library exports 16 types of vector, 14 types of rectangular matrix and 8 types of symmetric matrix[2].
They can be divided into two main categories: the *variable-size* containers, such as `DoubleVec`, `DoubleMat`, or `DoubleSym`, and the *fixed-size* containers, such as `DoubleVec3`, `DoubleMat2x2` or `DoubleSym2`.

☞ Following the C usage, all these math containers are *zero based*: a vector with size N extends from index 0 to N-1.

Variable-size containers and fixed-size containers

Besides the fact that only the containers of the first category can be resized - automatically or manually - they differ by the way the copy constructors and the copy operators work. The variable-size containers are merely *views* to data arrays, whereas the fixed-size containers actually hold the data as a member. The first category only holds a reference, and a copy implies only a copy of that reference, not the data (*shallow copy*). A copy of a container of the second category actually copies the data (*deep copy*).

Example:

```
DoubleVec   V1;             // Empty vector.
DoubleVec   V2(10, +1.0);   // Vector of 10 values, all intitialized to 1.0

V1.push_back (3.0)          // V1.size() == 1
V1.push_back (2.0)          // V1.size() == 2

V1 = V2;                    // Shallow copy[3] (V1.size() == 10).
                            // Previous values of V1 are lost (3.0 and 2.0).

V1[0] = 0.0;                // V2[0] == 0.0 (because the data are shared).
V2.clear();                 // V2.size() == 0 but V1.size() == 10
                            // The data are not deleted because still viewed
                            // from V1.
```

[1] *math1.html or math1.chm.*
[2] *Some other types are defined but not exported (they cannot be exchanged between dynamic libraries, only between statically linked entities): `vector_fixed<T,N>`, `matrix_fixed<T,M,N>`, `dense1D<T>`, `dense2D<T>` and `symmetric_full<T>`.*
[3] *A deep copy can be obtained with the template function `vecvec::copy` :*

```
   V1.resize (V2.size());     // Resize V1 to V2.size().
   vecvec::copy (V2, V1);     // Copy all V2 values into V1.
```

or with the "copy" member:

```
   V1.copy (V2);              // Resize V1 to V2.size() and copies the data.
```

Vectors	Rectangular matrices	Symmetric matrices
DoubleVec	DoubleMat	DoubleSymMat
FloatVec	FloatMat	FloatSymMat
IntVec	IntMat	IntSym
UIntVec	UIntMat	UIntSym
DoubleVec2		
DoubleVec3	DoubleMat3x1	DoubleSym2
DoubleVec4	DoubleMat2x2	
DoubleVec6	DoubleMat3x2 DoubleMat2x3	DoubleSym3
DoubleVec8		
DoubleVec9	DoubleMat3x3	
UIntVec2		
UIntVec3	UIntMat3x1	UIntSym2
UIntVec4	UIntMat2x2	
UIntVec6	UIntMat3x2 UIntMat2x3	UIntSym3
UIntVec8		
UIntVec9	UIntMat3x3	

Table 1 – Vectors and matrices exported by the `math1` library.

Views of the variable-size containers

Several variable-sized containers can have a view on the same array of data, but the view can be different from each other. The beginning and the size in the array are specific to each container. For instance in an array of 30 items, a first vector views items from 0 to 9 and a second one views items from 5 to 20.

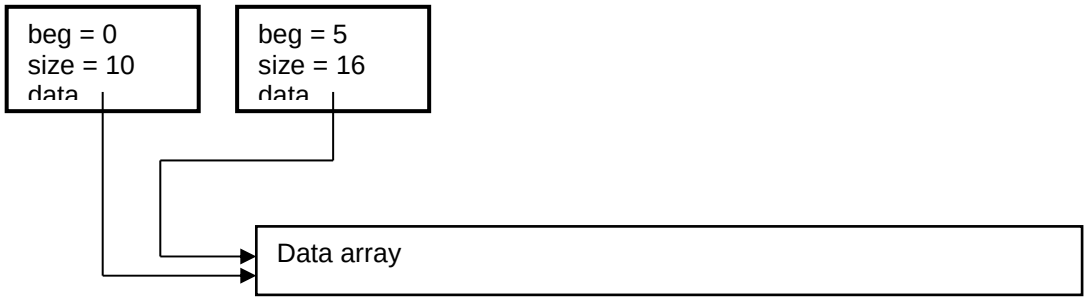


Figure 1 – Views and data in variable-sized vectors.

Elements from 5 to 9 are accessible from the two vectors.
When a destructor is called on a variable-size container, the data are destroyed only when no other container shares them anymore. A *smart pointer* mechanism is used to count the number of references on the data and the deallocation actually occurs when the count reaches null. The memory management is automatic (automatic garbage collection).

Example:

```
DoubleVec  *V1 = new DoubleVec(10, -1.0);
DoubleVec  *V2 = new DoubleVec(*V1);    // Shallow copy (share the data).

delete V1;   // The data are still referenced by V2.
delete V2;   // Now the data are destroyed too.
```

Fixed-size containers

The fixed-size math containers are deep-copy containers. The copy-constructor and the copy-operator do not make the data to be shared anymore but leads to actually different arrays in memory. They are simpler than the variable-size containers, and faster for short arrays, whereas the variable-size containers are more suited for big arrays.

Example:

```
DoubleVec2   V1;           // Vector of 2 unintitialized values (double).
DoubleVec2   V2(+1.0);     // Vector of 2 values intitialized to 1.0

V1[0] = 0.0;
V1[1] = -1.0;

V1 = V2;                   // Deep copy: V1 = {1, 1}, V2 = {1, 1}.
V2[0] = 0.0;               // V1 = {1, 1}, V2 = {0, 1}
V1[1] = 0.0;               // V1 = {1, 0}, V2 = {0, 1}
```

STL-like iterators and the generic math library

All these math containers are optimized for numerical computation.
The vector containers - variable-size and fixed-size - are equipped with STL-like iterators `begin()` and `end()`, to make them compatible with most of the STL algorithms. They also have access operators such as `operator[]`, and the usual functions for a vector class: `size()`, `empty()`, `front()`, `back()`...
The variable-size vectors are also equipped with members such as `reserve`, `resize`, `push_back`, and `pop_back`.
Aside from the STL algorithms, one can also use the math-specific template functions of the `vecscal`, `vecvec`, `matscal`, `matvec` and `matmat` libraries (cf. math1 API).

Example:

```
DoubleVec    V1(3), V2(3,-1.0);
DoubleMat    M(3, 10, 0.0);       // Matrix of doubles 3 by 10 set to 0.

vecvec::copy (V2, V1);            // Hard copy of V2 into V1 (sizes match).
vecvec::axpy (2.0, V1, V2);       // V2 += 2 * V1
vecscal::mult (-1.0, V1);         // V1 = -V1
```

The variable-size matrix types (`DoubleMat`, `IntMat` and `UIntMat`) have a behavior similar to the vectors with respect to the memory management and the copy operators (and the copy-constructors). In addition, the rows and the columns are equipped with iterators, just like the vectors, and the same template functions can be used on them.

Example:


```
const unsigned N  = 10;
const double   PI = 3.14159;
DoubleMat      pos(2, N);         // Uninitialized 2xN matrix.
DoubleVec2     V;                 // Uninitialized vector of size 2.

// Points on a circle.
for (unsigned j = 0; j < N; ++j)
{
   pos(0,j) = ::cos(j*2*PI/N);
   pos(1,j) = ::sin(j*2*PI/N);
}

// Set radius to 3.0
matscal::mult (3.0, pos);

// Copy column #2 to V (ok, dimensions match).
vecvec::copy (pos.col(2), V);

// Copy V to column #9 (the last one).
vecvec::copy (V, pos.col(9));

// Copy column #9 to column #0
vecvec::copy (pos.col(9), pos.col(0));

// Append V in a new column of pos (dimensions match).
pos.push_back (V);              // pos.cols() == 11 after this line.
```

Bound checking

In debug mode (with macro `_DEBUG` defined), bound violations abort the program. In release mode however, for best performance no check is performed and the user must take care not to out value the limits of the vectors and matrices.

Interoperability with other math containers

The API of the MeshTools library use exclusively vectors and matrices of the math1 library (such as `DoubleMat`, `UIntMat`, `FloatVec`...). To use the meshers with other types of vectors and matrices, the variable-size containers are equipped with constructors with raw pointers as arguments. Hence, they can view the data in any other math containers as long as the latter provide a way to get a raw pointer to their data, and that these data are *contiguous* in memory.
Remember that the variable-size containers implement *shallow copies*. This means that the arrays are shared, not copied. Therefore the memory management becomes a point to take care of and the user must keep in mind which library is responsible for deleting the memory upon exit. The default is that the allocator of the array remains responsible for its deletion.

Example:

```
double                buff1[100];                // Static raw C array.
double*               buff2 = new double[100];   // Dynamic raw C array.
std::vector<double>   buff3(100);                // STL vector.

DoubleVec   V1(50, buff1);         // Views the first 50 elements in buff1.
DoubleVec   V2(50, buff2);         // Views the first 50 elements in buff2.
DoubleVec   V3(50, buff3.begin()); // Views the first 50 elements in buff3.

V2.clear();          // buff2 is not deallocated.
V2 = V1;             // buff2 is not deallocated.
V3.clear();          // buff3 is not deallocated.
delete[] buff2;      // Dangerous but correct because buff2 is no longer
                     // referenced by any DoubleVec.
buff3.clear();       // Dangerous: data in buff3 may have been deallocated.
                     // Don't use V3 anymore.
```

Note that this can be done with the fixed-size containers of math1:

```
DoubleVec3  V3(1.0, 0.0, -1.0);
DoubleVec   V(3, V1.begin());     // Views the elements in V3.
```

The matrices of math1 can view also the data in an external container. In addition to the raw pointer to the data, the user must provide the number of rows, the number of columns and the stride between two columns (so-called *leading* dimension):

```
unsigned*   buff = new unsigned[30];
UIntMat     M(3, 10, /*ld=>*/ 3, buff);
```

As before, the matrix is not responsible for the deletion of the underlying buffer[4].

In the case where a container constructed this way is subsequently resized, it may "point" to another array of memory but the initial buffer remains valid:

```
double*     buff = new double[6];
DoubleVec   V(5, buff);

V.push_back (2.0);   // Reallocation and copy performed.
                     // buff is still alive, but V does not "point" to
                     // it anymore.
V.clear_hard();      // The new array of V is deallocated, not buff.
```

☞ As a rule of thumb, the lifetime of the external buffer must span the lifetime of the math1 container.

```
double*     buff = new double[6];

{
   DoubleVec    V(5, buff);
   ...    // Use V here.
}  // V is killed here but the buffer is spared.

delete[] buff;  // So long with buff.
```

We have seen how to construct math1 variable-size container upon other containers or buffers. To do the other way, we use the `data()` or `begin()` members to access the underlying data:

Example:

[4] *A default parameter "protect" in the constructors can be used to change this behaviour.*


```
DoubleVec     V(50, 0.0);
double*       buff = V.data();
unsigned      N    = V.size();     // Equals to 50

for (unsigned i = 0; i < N; ++i, ++buff)
   *buff = double(i);

assert (V[10] == 10.);             // Changes in buff have been seen in V.
```

```
DoubleMat     P(3, 40);
double*       buff = P.data();
unsigned      M    = P.rows();     // Equals to 3
unsigned      N    = P.cols();     // Equals to 40
unsigned      LD   = P.ld();       // Equals to 3 (here stride == rows).
```

```
for (unsigned j = 0; j < N; ++j)
   for (unsigned i = 0; i < M; ++i)
      buff[i + j*LD] = double(i + j*LD);
```

```
assert (P(0,10) == 30.);           // Changes in buff have been seen in P.
```

Here, the math1 vectors and matrices are responsible for the deletion of their data:

```
DoubleMat     P(3, 40);
double*       buff = P.data();

delete[] buff;       // Don't do that !
P.resize (3, 80);    // Crash now, or maybe later...
```


II – TUTORIAL

Before meshing a 2-D domain, the first step is to generate a 1-D mesh of the external contour. This chapter mostly details cases where the boundary mesh is obtained using some simple CM2 MeshTools functions. One example illustrates the case where the boundary mesh has been generated by other means and is simply read from a file.

Each example starts with including the file `stdafx.h` (can be a precompiled header), giving access to the classes and the functions of the library (API).

The general namespace `cm2` has nested namespaces such as `vecscal`, `vecvec`, `meshtools` or `triamesh`. The user can add "using namespace" directives in this `stdafx.h`. Keeping namespaces in the user's source code can however be useful to improve the legibility and to avoid name conflicts.

File "stdafx.h" [5]:

```
// MESHTOOLS
#include "meshtools.h"      // General purpose mesh routines
#include "meshtools1d.h"    // To generate 1D meshes

// MESHERS
#include "triamesh.h"       // TriaMesh mesher
#include "quadmesh.h"       // QuadMesh mesher

using namespace cm2;        // Main cm2 namespace can now be omitted.
```

Required libraries[6]:

```
cm2math1
cm2misc
cm2meshtools
cm2meshtools1d
cm2meshtools2d
cm2triamesh
cm2quadmesh
```

[5] If neither meshtools nor CM2 QuadMesh is used, the file "stdafx.h" can reduce simply to:
#include "triamesh.h"
and the required libraries are cm2math1, cm2misc, cm2meshtools, cm2meshtools2d and cm2triamesh
[6] On Windows, the lib names currently end with `_Win32_45` or `_x64_45`. For instance `cm2math1_x64_45.dll`. On Windows, file extensions for the libraries are .lib and .dll. On Linux/Unix/Mac OS platforms, file extensions are usually .a (static archive), .so or .dylib (dynamic lib).

II-1 GETTING STARTED – A SIMPLE SQUARE

This first example is a regular mesh of a square. The four boundary segments are equally discretized with 10 elements.

```
#include "stdafx.h"

// Display handler (optional).
static void display_hdl (void*, unsigned, const char* msg) { cout << msg; }


int main()
{
   const double       L = 10.0;
   const unsigned     N = 10;
   const DoubleVec2   P0(0.,0.), P1(L,0.), P2(L,L), P3(0.,L);
   DoubleMat          pos;
   UIntVec            indices;
   UIntMat            connectB;

   // UNLOCK THE DLL.
   triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   // VERTICES AND LINE MESHES.
   pos.push_back (P0);
   pos.push_back (P1);
   pos.push_back (P2);
   pos.push_back (P3);
   meshtools1d::mesh_straight (pos, 0, 1, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 1, 2, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 2, 3, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 3, 0, N, indices);
   meshtools1d::indices_to_connectE2 (indices, connectB);

   // THE 2D MESH.
   triamesh::mesher               the_mesher;
   triamesh::mesher::data_type    data (pos, connectB);
   the_mesher.run (data);

   // SOME OUTPUT INFO (OPTIONAL).
   data.print_info (&display_hdl);

   // VISUALISATION (OPTIONAL).
   meshtools::medit_output ("out.mesh", data.pos, data.connectM, CM2_FACET3);

   return 0;
}   // main
```

The resulting mesh is shown Figure 2.

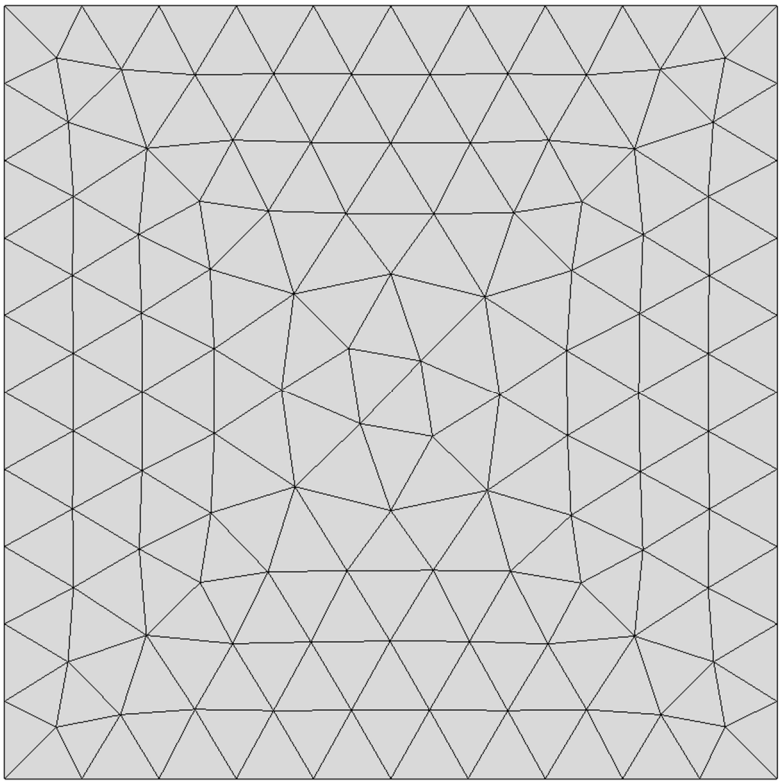


Figure 2 – Triangle mesh of a square.

Let us explain this program line by line.

Authorization of the library

The libraries `triamesh` and `quadmesh` are protected and need to be unlocked with a call to `triamesh::registration` or `quadmesh::registration`. Two strings must be provided for each library: the name of your company or organization that has acquired the license and a secret code - contact license@computing-objects.com for any licensing enquiry. Note that both strings are case sensitive and the registration call must be made each time the library is loaded into memory and *before* any run of the mesher.

```
triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");
```

Contour mesh

This is usually the heaviest part of the work for the user. In this example, we use only routines from the MeshTools libraries but the user is free to generate this contour mesh with any other tool or even to read it form a file[7]. Anyway, the 2-D meshers need this contour mesh as a couple of matrices: the matrix `pos` containing the points' coordinates and the connectivity matrix `connectB` of the boundary edges.

First, the corners of the square are created as four pair of coordinates in the `pos` matrix:

```
pos.push_back (P0);
pos.push_back (P1);
pos.push_back (P2);
pos.push_back (P3);
```

[7] *See example II-5.*

The `push_back` function appends a new column at the end of a matrix. The size of the column must match the current number of rows of the matrix. If the matrix is empty, the first vector sets this number of rows.
After these four push-backs, the dimensions of the `pos` matrix are 2x4.

Now that the four corners are present, we can create the points in between and the associated edges:

```
meshtools1d::mesh_straight (pos, 0, 1, N, indices); indices.pop_back();
meshtools1d::mesh_straight (pos, 1, 2, N, indices); indices.pop_back();
meshtools1d::mesh_straight (pos, 2, 3, N, indices); indices.pop_back();
meshtools1d::mesh_straight (pos, 3, 0, N, indices);
```

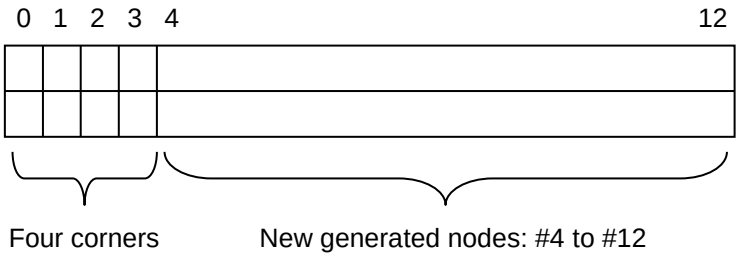
The `mesh_straight` routine of the `meshtools1d` library generates N-1 new points equally spaced into new appended columns in the `pos` matrix:

```
meshtools1d::mesh_straight
   (DoubleMat& pos, unsigned i0, unsigned i1, unsigned N, UIntVec& indices);
```

The index of each point, i.e. the column in matrix `pos`, is also appended to the vector `indices`. With i0 = 0 and i1 = 1, this vector contains upon exit of this function:

```
[0 4 5 6 7 8 9 10 11 12 1]
```

And the matrix `pos` is now of size 2x13:



The last value in the `indices` vector, i.e. value 1, must be suppressed to avoid having it twice:

```
indices.pop_back();
```

The second call to `mesh_straight` with i0 = 1 and i1 = 2 sets the `indices` vector to:

```
[0 4 5 6 7 8 9 10 11 12 1 13 14 15 16 17 18 19 20 21 2]
```

After the four line meshes, the matrix `pos` is of size 2x40 and the `indices` vector has 41 values - the last index equals to the first, here zero, to indicate that the contour is closed[8].

The vector of indices is used to create the connectivity matrix of the boundary mesh:

[8] *The same result could have been achieved with:*

```
UIntVec    hard_nodes(5);
hard_nodes[0] = 0;
hard_nodes[1] = 1;
hard_nodes[2] = 2;
hard_nodes[3] = 3;
hard_nodes[4] = 0;
meshtools1d::mesh_straight (pos, hard_nodes, 4*N, indices);
```

This variant of `mesh_straight` meshes a polyline going through some constrained points (`hard_nodes`)

```
meshtools1d::indices_to_connectE2 (indices, connectB);
```

The `connectB` matrix has now dimensions 2x40:

```
2x40 [0 4 5 6 7 ... 39
      4 5 6 7 8 ...  0]
```

Now we have done the boundary mesh, all we have to do is to call the 2-D mesher. This done by creating a data structure holding this 1D mesh and make the mesher run on it:

```
triamesh::mesher::data_type    data (pos, connectB);
the_mesher.run (data);
```

Upon exit, the matrix `data.pos` is bigger and contains all the new points generated inside the square by the 2-D mesher. These new points are appended to the original matrix. The initial 40 points are still untouched in the first 40 columns.
The connectivity of the final mesh is stored in the matrix `data.connectM`, each column storing the indices of the nodes for an element[9]. `connectM(i,j)` is the i^{th} local node of the j^{th} element.

Printed information about the generated mesh and a MEDIT[10] output file are obtained with:

```
data.print_info (&display_hdl);
meshtools::medit_output ("out.mesh", data.pos, data.connectM, CM2_FACET3);
```

Here is the output given by `data.print_info (&display_hdl)`:

[9] *The elements are always oriented counter-clock wise (director up)*
[10] *MEDIT is a free vizualisation program (http://www.ann.jussieu.fr/~frey/logiciels/medit.html). Other output formats are: Ensight, FEMAP (neutral), Nastran, STL, VTK and Wavefront OBJ.*


```
**************************************
************* CM2 TRIAMESH ************
**************************************
Version         : 4.6.0.0
Hard nodes      : 40 (40 in)
Hard edges      : 40 (40 in)
Nodes           : 132
Triangles       : 222
Subdomains      : 1
Area            : 1.600000E+001
Qmin            : 8.374821E-001
Front time      : 0.00 s
Refine time     : 0.00 s
Optim time      : 0.00 s
Total time      : 0.00 s (INF t/s)

************ HISTOGRAM QS ************
Total number of bins     :           11
Total number of counts   :          222
Number of larger values  :            0
Number of smaller values :            0
V max                    : 9.990807E-001
V mean                   : 9.462349E-001
V min                    : 8.374821E-001

Bin number        -- Bin boundaries --         Hits

   10            0.90             1.00          167
    9            0.80             0.90           55
    8            0.70             0.80            0
    7            0.60             0.70            0
    6            0.50             0.60            0
    5            0.40             0.50            0
    4            0.30             0.40            0
    3            0.20             0.30            0
    2            0.10             0.20            0
    1            0.01             0.10            0
    0            0.00             0.01            0
```

Figure 3 – Print info for the square example.

The generated mesh has 132 nodes and 222 triangles, for an initial contour mesh of 40 nodes and 40 edges (hard nodes and hard edges). The times spent in the three steps of the meshing process (front, refine, optimize) are given in seconds[11]. The front mesh is the triangulation mesh with only the boundary hard nodes. In the second step, new nodes are generated inside the domain to get elements with good shape and size. Finally, the last step is for geometrical and topological optimizations to improve the quality of the elements.

The formula used to compute the shape quality of a triangle writes:

$$Q_s = 4\sqrt{3}\,\frac{S}{L_{\max}\,P}$$

with:

[11] *Here the times are below 0.01 s. All runs are done with x64 CM2 libs (VS 2010 MD build) on Windows® 8 x64 with Intel® Xeon® E3-1270 V2 3.5 GHz (1 thread, turbo boost disabled). The typical speed with default settings on such a platform ranges from 10 000 quads / s. (CM2 QuadMesh Aniso) to more than 300 000 triangles / s. (CM2 TriaMesh).*

S	Area of the triangle.
$L_{\max}$	Length of the longest edge of the triangle.
P	Perimeter of the triangle

This quality measure ranges from 0 for a degenerated triangle, to 1 for an equilateral triangle.
On the square example, the worst shape quality is 0.83 and the average is 0.94.

The *size quality* is also an important parameter to take into account. The size quality of an edge is a measure based upon its actual length and the target size values defined at its two vertices. A size quality of one indicates that the edge has the optimal length. A too short edge has a size quality lesser than one - but always positive -, and a too long edge has a size quality greater than one. For instance, an edge with a quality of two is twice as long as it should be.

The formula used to compute the length quality of an edge *AB* writes:

$$Q_h^{AB} = L_{AB} \frac{\ln\left(\frac{h_A}{h_B}\right)}{h_A - h_B}$$

with:

L_{AB}	Actual length of edge *AB*.
h_A	Target size at node *A* (expected edge length at *A*).
h_B	Target size at node *B* (expected edge length at *B*).

Let's introduce also at this point the *h-shock* measure of an edge:

$$hs^{AB} = \min\left(\frac{h_A}{h_B}, \frac{h_B}{h_A}\right)^{\frac{1}{Q_h^{AB}}} - 1$$

These two measures are dimensionless and positive.

When $h_A = h_B$ the h-shock is null and the length quality simply writes $Q_h^{AB} = \frac{L_{AB}}{h_A}$.

When $Q_h^{AB} = 1$ edge *AB* is considered having optimal length with repect to its target mesh sizes h_A and h_B.

To optimize a mesh we need to improve simultaneously both the shape quality of the elements and the size quality of the edges. Unfortunately, these two targets are often contradictory.

The histogram of the size qualities can be computed either inside the mesher by raising the flag `mode.compute_Qh_flag`[12] before meshing or with a posteriori call to the auxiliary function `meshtools::edge_qualities`.
On the square example, the size qualities are well centered on the value 1 with a small variance:

[12] *See § III-3 for full description of the options of the meshers.*


```
************ HISTOGRAM QH ************
Total number of bins     :           20
Total number of counts   :          353
Number of larger values  :            0
Number of smaller values :            0
V max                    : 1.284916E+000
V mean                   : 1.021125E+000
V min                    : 7.444225E-001

Bin number        -- Bin boundaries --         Hits

   19            1.90             2.00            0
   18            1.80             1.90            0
   17            1.70             1.80            0
   16            1.60             1.70            0
   15            1.50             1.60            0
   14            1.40             1.50            0
   13            1.30             1.40            0
   12            1.20             1.30           16
   11            1.10             1.20           36
   10            1.00             1.10          174
    9            0.90             1.00          102
    8            0.80             0.90           17
    7            0.70             0.80            8
    6            0.60             0.70            0
    5            0.50             0.60            0
    4            0.40             0.50            0
    3            0.30             0.40            0
    2            0.20             0.30            0
    1            0.10             0.20            0
    0            0.00             0.10            0
```

Figure 4 – Histogram of the size-qualities of all the edges in the square example.

In order to mesh with quadrangles, all is needed is to change the class of the mesher:

```
#include "stdafx.h"

// Display handler (optional).
static void display_hdl (void*, unsigned, const char* msg) { cout << msg; }


int main()
{
   const double       L = 10.0;
   const unsigned     N = 10;
   const DoubleVec2   P0(0.,0.), P1(L,0.), P2(L,L), P3(0.,L);
   DoubleMat          pos;
   UIntVec            indices;
   UIntMat            connectB;

   // UNLOCK THE DLL.
   quadmesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   // VERTICES AND LINE MESHES.
   pos.push_back (P0);
   pos.push_back (P1);
   pos.push_back (P2);
   pos.push_back (P3);
   meshtools1d::mesh_straight (pos, 0, 1, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 1, 2, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 2, 3, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 3, 0, N, indices);
   meshtools1d::indices_to_connectE2 (indices, connectB);

   // THE 2D MESH.
   quadmesh::mesher               the_mesher;
   quadmesh::mesher::data_type    data (pos, connectB);
   the_mesher.run (data);
   data.print_info (&display_hdl);

   // VISUALISATION.
   meshtools::medit_output ("out.mesh", data.pos, data.connectM, CM2_FACEQ4);

   return 0;
}   // main
```

In this particular case, the generated mesh is a perfect structured quad mesh with all qualities equal to 1[13].

[13] *We could get the same structured Q4 mesh with* `meshtools2d::mesh_struct_Q4`.

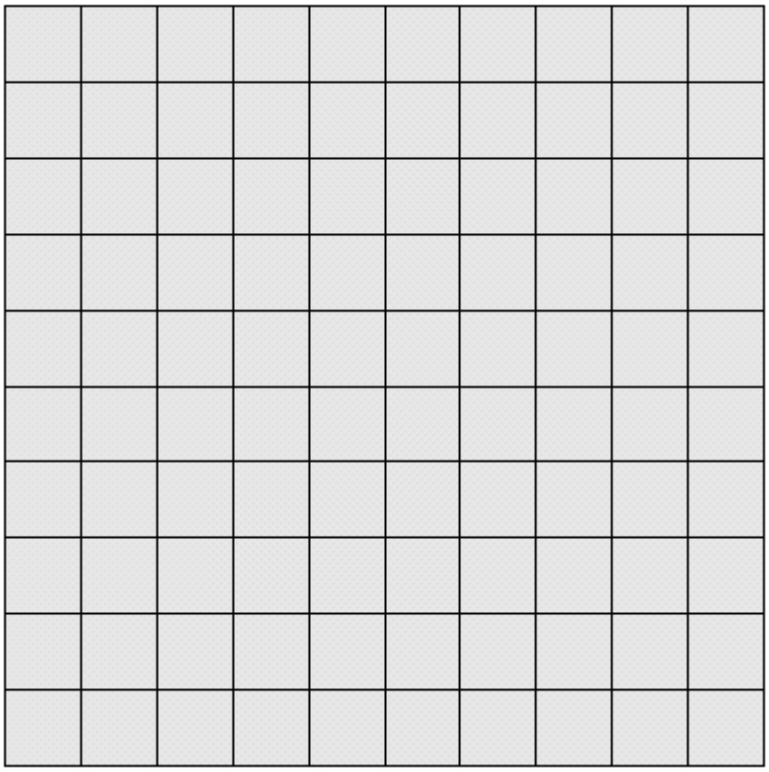


Figure 5 – Square meshed with quads.

For a plane quadrangle, we use the following measure of the shape quality:

$$Q_s = 8\sqrt{2}\,\frac{S_{\min}}{L_{\max}\,P}$$

with:

$S_{\min}$	Minimum area of the four triangles.
$L_{\max}$	Max length of the four sides and the two diagonals.
P	Perimeter of the quad.

This measure gives the maximal value 1 only for a square.

The size quality is given by the same measure as for the triangles (because it is based on edges only).

II-2 SQUARE WITH AN INTERNAL LINE

Starting from the previous example, we add a circle inside the square. Here is the program for a triangle mesh:

```
#include "stdafx.h"

int main()
{
   const double       L  = 10.0;
   const double       R  = 3.0;
   const unsigned     N1 = 10;
   const unsigned     N2 = 20;
   const DoubleVec2   P0(0.,0.), P1(L,0.), P2(L,L), P3(0.,L);
   const DoubleVec2   P4(L/2+R, L/2);
   DoubleMat          pos;
   UIntVec            indices;
   UIntMat            connectB;

   // UNLOCK THE DLL.
   triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   // VERTICES AND LINE MESHES.
   pos.push_back (P0);
   pos.push_back (P1);
   pos.push_back (P2);
   pos.push_back (P3);
   pos.push_back (P4);
   meshtools1d::mesh_straight (pos, 0, 1, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 1, 2, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 2, 3, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 3, 0, N, indices);
   meshtools1d::indices_to_connectE2 (indices, connectB);
   indices.clear();
   meshtools1d::extrude_rotate (pos, 4, DoubleVec2(L/2.,L/2.),
                                2.*M_PI, N2, indices);
   indices.back() = indices.front();
   meshtools1d::indices_to_connectE2 (indices, connectB);

   // THE 2D MESH.
   triamesh::mesher               the_mesher;
   triamesh::mesher::data_type    data (pos, connectB);
   the_mesher.run (data);

   // VISUALISATION.
   meshtools::medit_output ("out.mesh", data.pos, data.connectM, CM2_FACET3);

   return 0;
}   // main
```

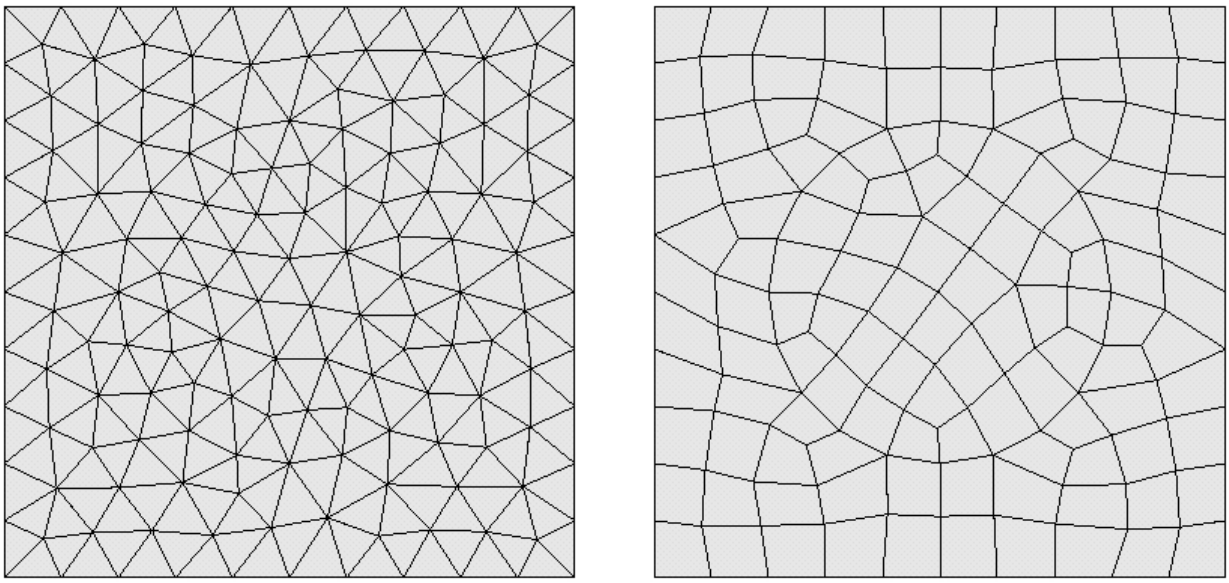


Figure 6 – Square with internal line (T3 and all-Q4).

The mesh of the circle is generated with the function `extrude_rotate` from the `meshtools1d` library. The rotation is defined by a center, here by the point `DoubleVec2(L/2,L/2)`, and a rotation scalar around Oz, here `2*M_PI`
The circular line is discretized using 20 elements[14] starting from point #4. Here, the last generated point - point #24 - is coincident with the first one - point #4. In order to topologically close the circle, it is important to replace value 24 with value 4 in the `indices` vector, so that the first and the last point are *identical*, not only coincident[15]:

```
indices.back() = indices.front();
```

As for the external contour, these indices are converted into edges with the `indices_to_connectE2` function and appended to the `connectB` matrix. The meshing algorithm makes the difference between external and internal boundary edges.

Again, to mesh with quads, we simply replace the `triamesh` namespace with `quadmesh`. Moreover, if we accept some triangles, we can get a better mesh. This is the *quad-dominant* mode:

[14] *Remember that CM2 QuadMesh needs an even number of edges on each line (external and internal lines) in all-quad mode.*
[15] *Note that the coordinates at column 24 in the `pos` matrix will remain unused.*


```
#include "stdafx.h"

int main()
{
   const double       L  = 10.0;
   const double       R  = 3.0;
   const unsigned     N1 = 10;
   const unsigned     N2 = 20;
   DoubleMat          pos;
   const DoubleVec2   P0(0.,0.), P1(L,0.), P2(L,L), P3(0.,L);
   const DoubleVec2   P4(L/2+R, L/2);
   UIntVec            indices;
   UIntMat            connectB;

   // UNLOCK THE DLL.
   triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   // VERTICES AND LINE MESHES.
   pos.push_back (P0);
   pos.push_back (P1);
   pos.push_back (P2);
   pos.push_back (P3);
   pos.push_back (P4);
   pos.push_back (DoubleVec2(L/2+R, L/2));
   meshtools1d::mesh_straight (pos, 0, 1, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 1, 2, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 2, 3, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 3, 0, N, indices);
   meshtools1d::indices_to_connectE2 (indices, connectB);
   indices.clear();
   meshtools1d::extrude_rotate (pos, 4, DoubleVec2(L/2.,L/2.),
                                2.*M_PI, N2, indices);
   indices.back() = indices.front();
   meshtools1d::indices_to_connectE2 (indices, connectB);

   // THE 2D MESH.
   quadmesh::mesher               the_mesher;
   quadmesh::mesher::data_type    data (pos, connectB);
   the_mesher.mode.all_quad_flag = false;
   the_mesher.run (data);

   // VISUALISATION.
   meshtools:medit_output ("out.mesh", data.pos, data.connectM,
                           CM2_FACE_MIX);

   return 0;
}   // main
```

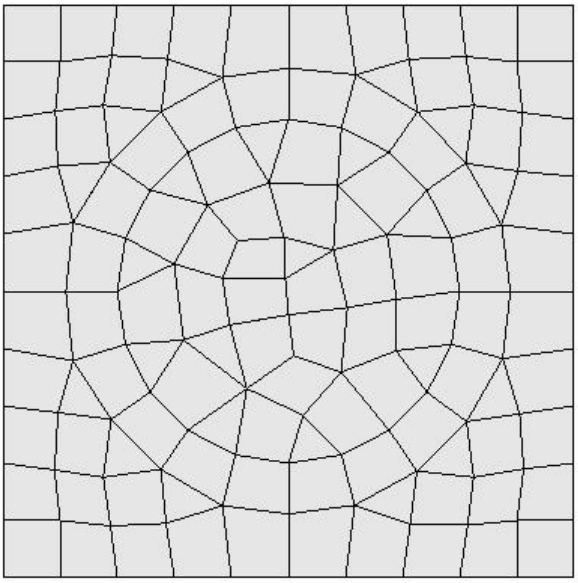


Figure 7 – Square with internal line (quad-dominant mode).

II-3 SQUARE WITH INTERNAL HOLES

A hole is an internal closed contour with edges oriented the opposite way from the external contour. Note that this implies that all edges of the external contour should be oriented in a uniform way (either clockwise or counter-clockwise)[16]. Based on the previous example, we simply change the sign of the rotation vector to revert the orientation of the internal edges and thus to remove the disk from the domain:

```
meshtools1d::extrude_rotate (pos, 4, DoubleVec3(L/2.,L/2.),
                             -2.*M_PI, N2, indices);
```

And the resulting meshes give:

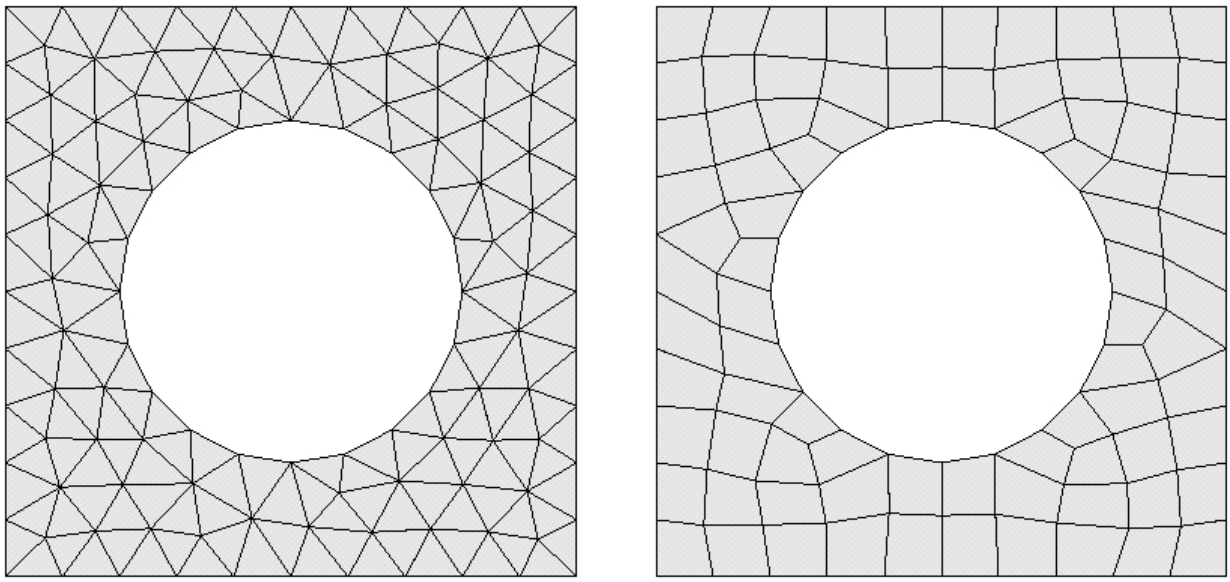


Figure 8 – Square with a circular hole (T3 and all-Q4).

One can nest alternatively positive and negative rotations:

[16] *Without any hole, the orientation of the external contour is irrelevant.*

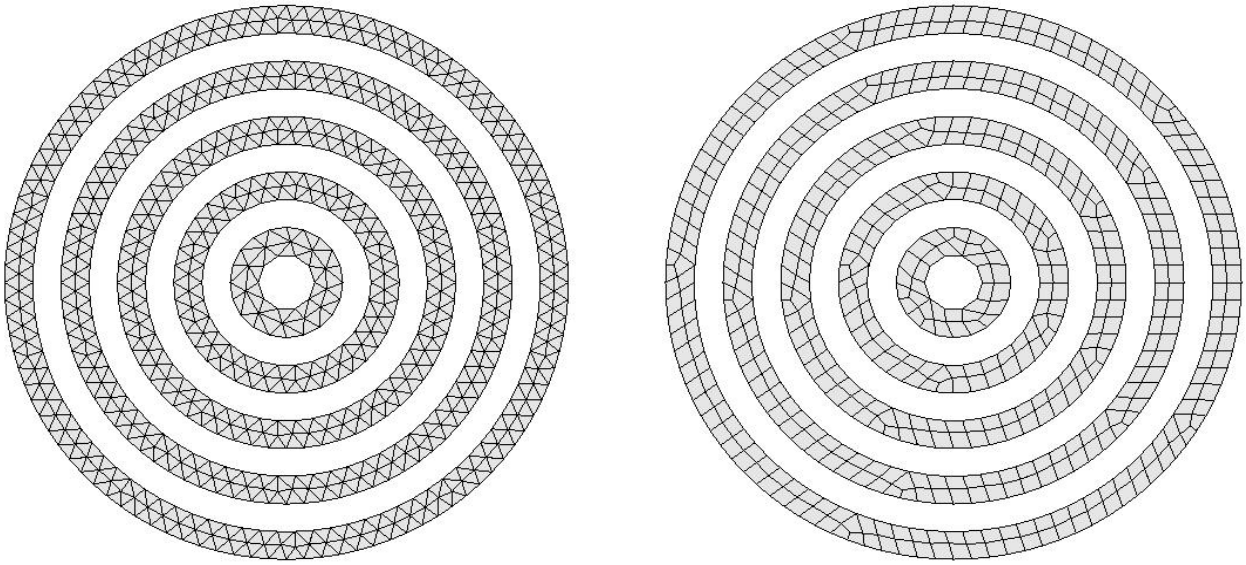


Figure 9 – Concentric circles with alternate orientation (T3 and all-Q4).

II-4 SQUARE WITH GRADING MESH SIZE

There are two ways to get a graded size in a mesh. First, you can simply generate edges with varying size on the boundary or interior line. The mesher computes a size chart on each hard node[17]. It interpolates these values inside the domain and generates elements accordingly.
To illustrate this, let us use again the example of the square. Instead of meshing regularly the four segments of the contour, we will specify different target sizes on each four vertices:

```
#include "stdafx.h"

int main()
{
   const double       L = 10.0;
   const unsigned     N = 10;
   const DoubleVec2   P0(0.,0.), P1(L,0.), P2(L,L), P3(0.,L);
   DoubleMat          pos;
   UIntVec            indices, hard_nodes(5);
   DoubleVec          sizes(5);
   UIntMat            connectB;

   // UNLOCK THE DLL.
   triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   // VERTICES AND LINE MESHES.
   pos.push_back (P0);
   pos.push_back (P1);
   pos.push_back (P2);
   pos.push_back (P3);
   hard_nodes[0] = 0;  sizes[0] = 0.1*L/N;
   hard_nodes[1] = 1;  sizes[1] = 2.0*L/N;
   hard_nodes[2] = 2;  sizes[2] = 0.1*L/N;
   hard_nodes[3] = 3;  sizes[3] = 2.0*L/N;
   hard_nodes[4] = 0;  sizes[4] = 0.1*L/N;
   meshtools1d::mesh_straight (pos, hard_nodes, sizes, true, indices);
   meshtools1d::indices_to_connectE2 (indices, connectB);

   // THE 2D MESH.
   triamesh::mesher              the_mesher;
   triamesh::mesher::data_type   data (pos, connectB);
   the_mesher.run (data);

   // VISUALISATION.
   meshtools::medit_output ("out.mesh", data.pos, data.connectM, CM2_FACET3);

   return 0;
}   // main
```

This variant of the `mesh_straight` function uses a vector of hard nodes, like a polygonal line, and a vector of target size values, one value for each hard node. The contour mesh is generated to fit best the target values on the four corners.
These target sizes are not used by the 2-D mesher. Only the resulting edge lengths of the contour will be used to compute the 2-D size field.

[17] *By averaging the lengths of the adjacents edges to each hard node.*

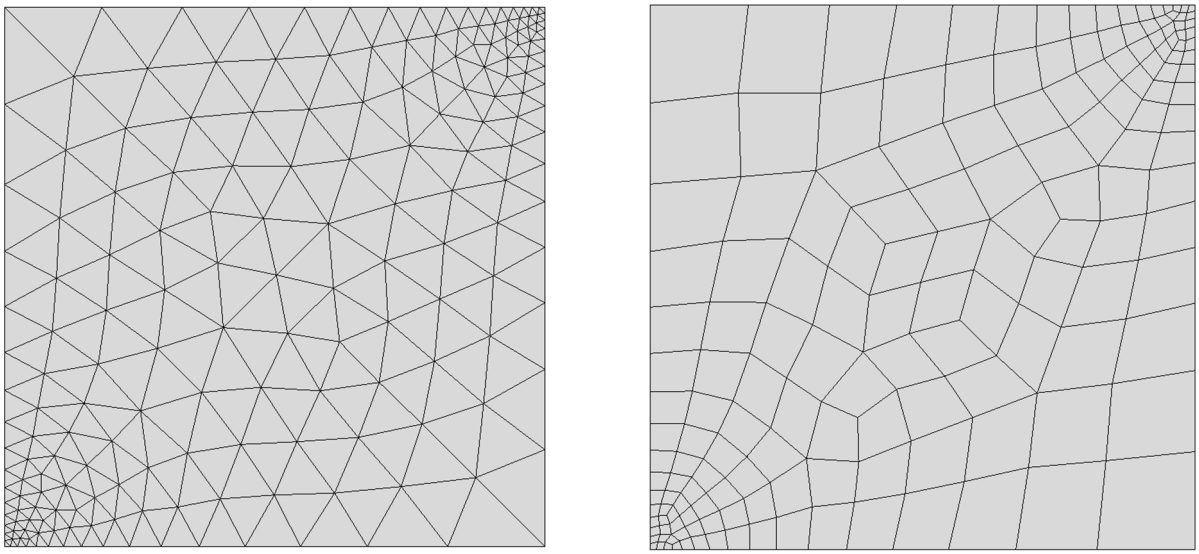


Figure 10 – Meshes with grading size (T3 and all-Q4).

The second way to get grading sizes is to specify manually, in the data of the 2-D mesher, the target size values on some hard nodes. This is explained in the next section.

II-5 SQUARE WITH AN INTERNAL HARD NODE

Until now, we have seen only three fields of the structure used to exchange data with the mesher:

- The `pos` matrix for the coordinates of the points.
- The `connectB` matrix for the connectivity of the hard edges.
- The `connectM` matrix for the connectivity of the 2-D mesh.

In this example, we will add an isolated hard node at the center of the square and specify a target size for the elements near this node. This will be done using the two new fields `isolated_nodes` and `metrics`:

```
#include "stdafx.h"

int main()
{
   const double       L = 10.0;
   const unsigned     N = 10;
   const DoubleVec2   P0(0.,0.), P1(L,0.), P2(L,L), P3(0.,L), P4(L/2.,L/2.);
   DoubleMat          pos;
   UIntVec            indices;
   UIntMat            connectB;

   // UNLOCK THE DLL.
   triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   // VERTICES AND LINE MESHES.
   pos.push_back (P0);
   pos.push_back (P1);
   pos.push_back (P2);
   pos.push_back (P3);
   pos.push_back (P4);
   meshtools1d::mesh_straight (pos, 0, 1, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 1, 2, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 2, 3, N, indices); indices.pop_back();
   meshtools1d::mesh_straight (pos, 3, 0, N, indices);
   meshtools1d::indices_to_connectE2 (indices, connectB);

   // THE 2D MESH.
   triamesh::mesher               the_mesher;
   triamesh::mesher::data_type    data (pos, connectB);
   data.isolated_nodes.push_back (4);
   data.metrics.resize (5, 0.0);
   data. metrics[4] = 0.1*L/N;
   the_mesher.run (data);

   // VISUALISATION.
   meshtools::medit_output ("out.mesh", data.pos, data.connectM, CM2_FACET3);

   return 0;
}   // main
```

We have created a new point at the center with coordinates placed in column #4 of matrix `pos`. Its index, i.e. 4, is then pushed into the vector `data.isolated_nodes`. This new field serves to store all the isolated nodes that must be present in the final mesh.

The vector `data.metrics` stores the user-specified target sizes. If the value for a node is zero - or negative or not present -, the automatically computed value will be used instead[18].
In our example, the vector is resized to 5 with all values set to zero except for point #4 where we ask for a 10 times finer mesh around it.

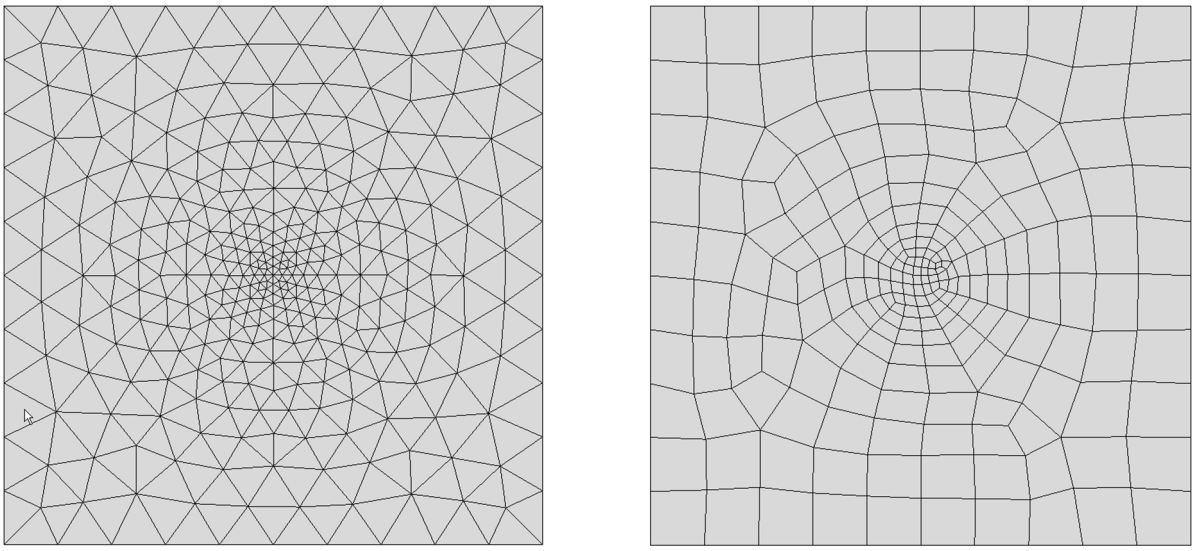


Figure 11 – Mesh concentration near a hard node (T3 and all-Q4).

[18] *For an isolated node, the computed size is based on the size value of the nearest nodes.*

II-6 MULTIPLE MESHES

The matrix `connectB` can contain several internal lines. It can also contain several external contours. This means that several disconnected domains can be meshed simultaneously. Some care must be taken however in the orientation of these contours. For multiple domains, the edges of all external contours must be oriented the same way, for instance counter-clockwise (the so-called *positive* orientation). In addition, these contours must not intersect each other.

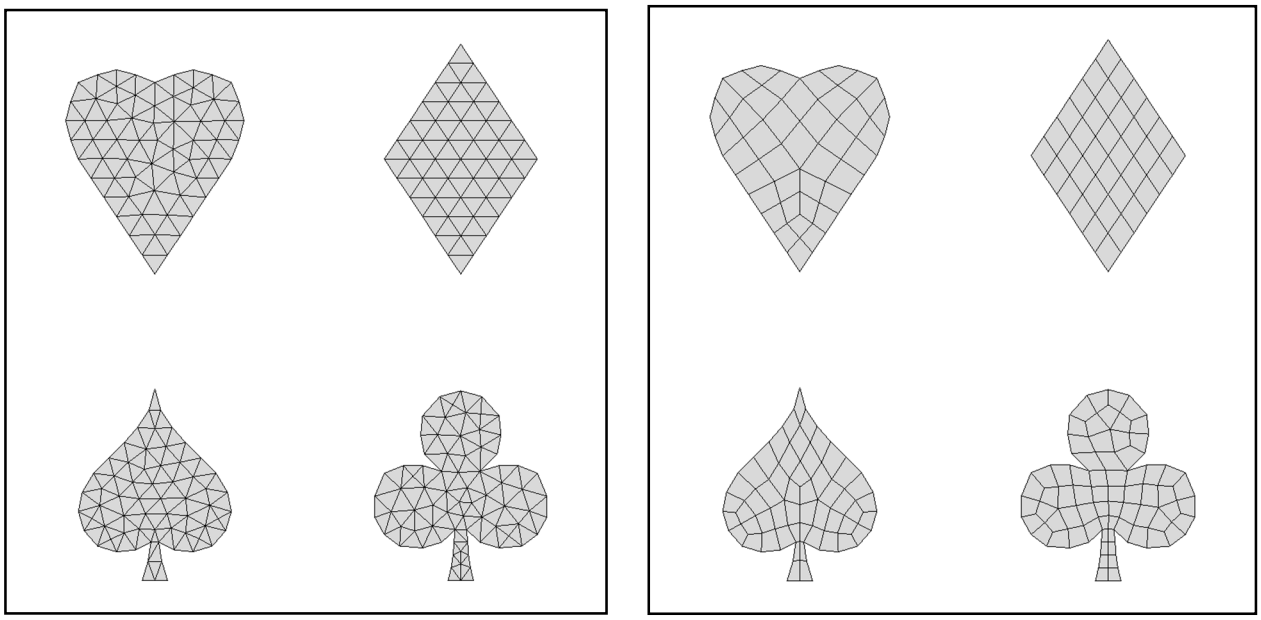


Figure 12 – Multiple meshes (T3 and all-Q4).
The four domains are meshed simultaneously.

In this example, the coordinates matrix and the connectivity of the contour meshes are read from a file[19]:

[19] *We could also have used the function* `meshtools1d::mesh_spline` *which generate 1D meshes along splines.*

```
#include "stdafx.h"

int main()
{
   ifstream                      istrm ("cards.dat");
   triamesh::mesher              the_mesher;
   triamesh::mesher::data_type   data;

   triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   matio::read (istrm, data.pos);
   matio::read (istrm, data.connectB);

   the_mesher.run (data);
   meshtools::medit_output ("out.mesh", data.pos, data.connectM, CM2_FACET3);

   return 0;
}   // main
```

The input boundary meshes are read from an ASCII file with `matio::read`[20].
The format for the matrices is:

```
n X m [
d0,0   d0,1   d0,2  ... d0,m-1
d1,0   d1,1   d1,2  ... d1,m-1
...
dn-1,0 dn-1,1 dn-1,2... dn-1,m-1 ]
```

The format for each component of the matrix is free.

For instance a 2x4 `DoubleMat` can be stored as:

```
2 X 4 [
0 0.5 1  2.0
0  1  1  2.E-1]
```

Notes:

- We can see also in this example that the meshes may not be symmetric with a symmetric contour.
- In the all-quad case, we set indeed the flag `multi_structured_flag = true`. This is the reason why the diamond is meshed in a structured manner.

As an exercise, we can get the same result by making four successive meshes and concatenating the results:

[20] *A similar `matio::transpose_read` function can read a matrix and transpose it on the fly.*
This can be more useful because it is usually more convenient to store the transposed matrices in the ASCII files.


```
#include "stdafx.h"

int main()
{
   ifstream                     istrm
   UIntMat                      connectM;
   DoubleMat                    pos;

   triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   triamesh::mesher             the_mesher;
   triamesh::mesher::data_type  data;

   istrm.open ("heart.dat");
   matio::read (istrm, data.pos);
   matio::read (istrm, data.connectB);
   the_mesher.run (data);
   pos.push_back (data.pos);
   connectM.push_back (data.connectM);

   istrm.open ("spade.dat");
   matio::read (istrm, data.pos);
   matio::read (istrm, data.connectB);
   the_mesher.run (data);
   matscal::add (pos.cols(), data.connectM);   // Shift indices.
   pos.push_back (data.pos);
   connectM.push_back (data.connectM);

   istrm.open ("diamond.dat");
   matio::read (istrm, data.pos);
   matio::read (istrm, data.connectB);
   the_mesher.run (data);
   matscal::add (pos.cols(), data.connectM);   // Shift indices.
   pos.push_back (data.pos);
   connectM.push_back (data.connectM);

   istrm.open ("club.dat");
   matio::read (istrm, data.pos);
   matio::read (istrm, data.connectB);
   the_mesher.run (data);
   matscal::add (pos.cols(), data.connectM);   // Shift indices.
   pos.push_back (data.pos);
   connectM.push_back (data.connectM);

   meshtools::medit_output ("out.mesh", pos, connectM, CM2_FACET3);

   return 0;
}   // main
```


II-7 SHARED BOUNDARIES

Edges can be shared between some contours and lines. In this case, some edges are defined several times (usually twice) in the `connectB` matrix, but with different orientation. In addition, it is sometimes more convenient for the user to generate the 1D meshes of the contours independently from each other. That usually implies duplicated nodes on the shared contours.

The following example deals with such a case.
Consider three subdomains, all oriented counter-clockwise as defined below. Several edges are shared between subdomains, but with different orientation. We also want to mesh the contours of the subdomains independently from each other but without any duplicated nodes.

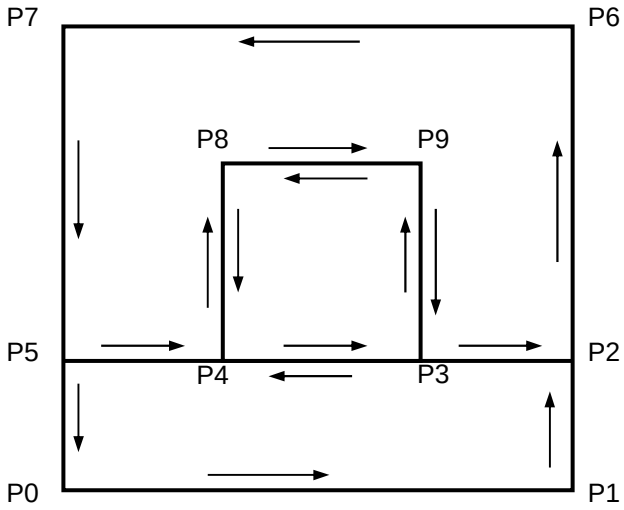


Figure 13 – Domain composed of three attached subdomains.

The shared edges are no problem for the mesher. The duplicate nodes however must be avoided. In non-strict mode (see § III-3), duplicated nodes can be discarded indeed but that implies also that the associated edges cannot be enforced. As a side effect, the mesher may not be able to tell the sign of the inner square, and that can lead to a hole.
The solution consists in merging the nodes after the meshing of the edges before the 2-D meshing:

```
#include "stdafx.h"

static void mesh_segment
  (DoubleMat& pos, UIntMat& connectB,
   unsigned start_index, unsigned stop_index, unsigned num_edges)
{
   UIntVec   indices;
   meshtools1d::mesh_straight
                     (pos, start_index, stop_index, num_edges, indices);
   meshtools1d::indices_to_connectE2 (indices, connectB);
}

int main()
{
   const DoubleVec2   P0(0, 0);
   const DoubleVec2   P1(10, 0);
   const DoubleVec2   P2(10, 2);
   const DoubleVec2   P3(8, 2);
   const DoubleVec2   P4(2, 2);
   const DoubleVec2   P5(0, 2);
   const DoubleVec2   P6(10, 10);
   const DoubleVec2   P7(0, 10);
   const DoubleVec2   P8(2, 8);
   const DoubleVec2   P9(8, 8);
   const unsigned     N = 4;
   UIntMat            connectB;

   // UNLOCK THE DLL.
   triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   // VERTICES POINTS.
   pos.push_back (P0);
   pos.push_back (P1);
   pos.push_back (P2);
   pos.push_back (P3);
   pos.push_back (P4);
   pos.push_back (P5);
   pos.push_back (P6);
   pos.push_back (P7);
   pos.push_back (P8);

   // BOTTOM RECTANGLE POSITIVE (I.E. COUNTER-CLOCKWISE).
   mesh_segment (pos, connectB, 0, 1, N);
   mesh_segment (pos, connectB, 1, 2, N);
   mesh_segment (pos, connectB, 2, 3, N);
   mesh_segment (pos, connectB, 3, 4, N);
   mesh_segment (pos, connectB, 4, 5, N);
   mesh_segment (pos, connectB, 5, 0, N);

   // TOP HORSESHOE POSITIVE (I.E. COUNTER-CLOCKWISE).
   mesh_segment (pos, connectB, 2, 6, N);
   mesh_segment (pos, connectB, 6, 7, N);
   mesh_segment (pos, connectB, 7, 5, N);
   mesh_segment (pos, connectB, 5, 4, N);
   mesh_segment (pos, connectB, 4, 8, N);
   mesh_segment (pos, connectB, 8, 9, N);
   mesh_segment (pos, connectB, 9, 3, N);
   mesh_segment (pos, connectB, 3, 2, N);

   // INNER SQUARE POSITIVE (I.E. COUNTER-CLOCKWISE).
   mesh_segment (pos, connectB, 3, 9, N);
   mesh_segment (pos, connectB, 9, 8, N);
```



```
   mesh_segment (pos, connectB, 8, 4, N);
   mesh_segment (pos, connectB, 4, 3, N);

   // MERGE TOGETHER DUPLICATED NODES.
   meshtools::merge (pos, connectB);

   // THE 2D MESH.
   triamesh::mesher               the_mesher;
   triamesh::mesher::data_type    data (pos, connectB);
   the_mesher.run (data);

   // VISUALISATION.
   meshtools::medit_output ("out.mesh", data.pos, data.connectM, CM2_FACET3);

   return 0;
}   // main
```

Note that this solution works because the shared edges are discretized similarly and the nodes are coincident.

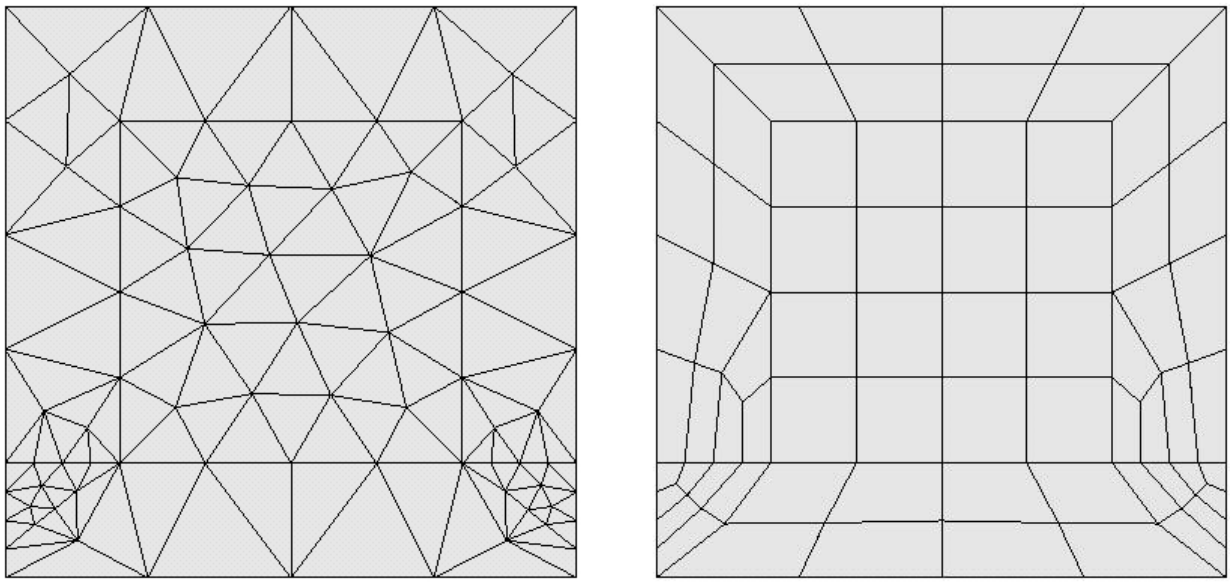


Figure 14 – Example with shared boundaries (T3 and all-Q4).

Note that the contour of the inner square is oriented completely both ways (positive and negative). In such a case, the mesher favors the positive orientation and meshes the inner square.

A similar case occurs when an inner contour is not properly oriented (see figure below). The mesher considers the inner domain to have the same status as the "most external domain" adjacent to it. Here the most external domain adjacent to the inner square is the outer square. Hence, the inner square will be meshed (i.e. no hole).

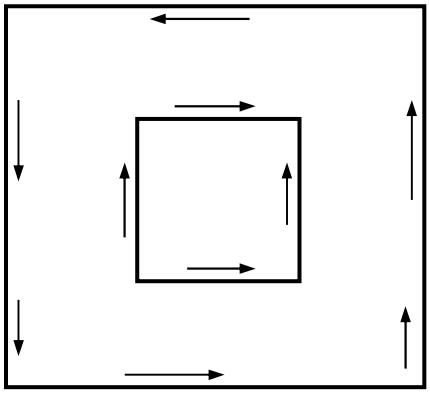


Figure 15 – Example of ambiguous orientation of an inner contour.

Here is another example where a hole is adjacent to the external contour. In this case, the most external domain adjacent to the inner square is the outside void. Hence, the inner square will not be meshed (i.e. hole).

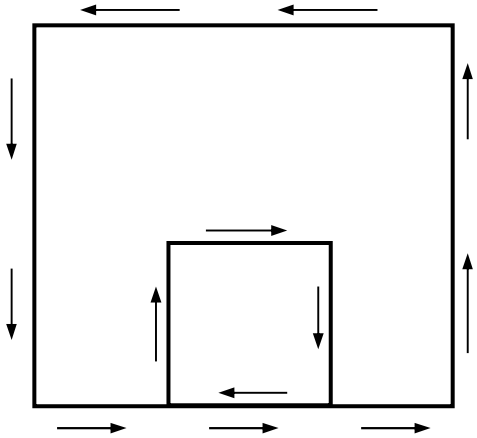


Figure 16 – Hole adjacent to the external contour.

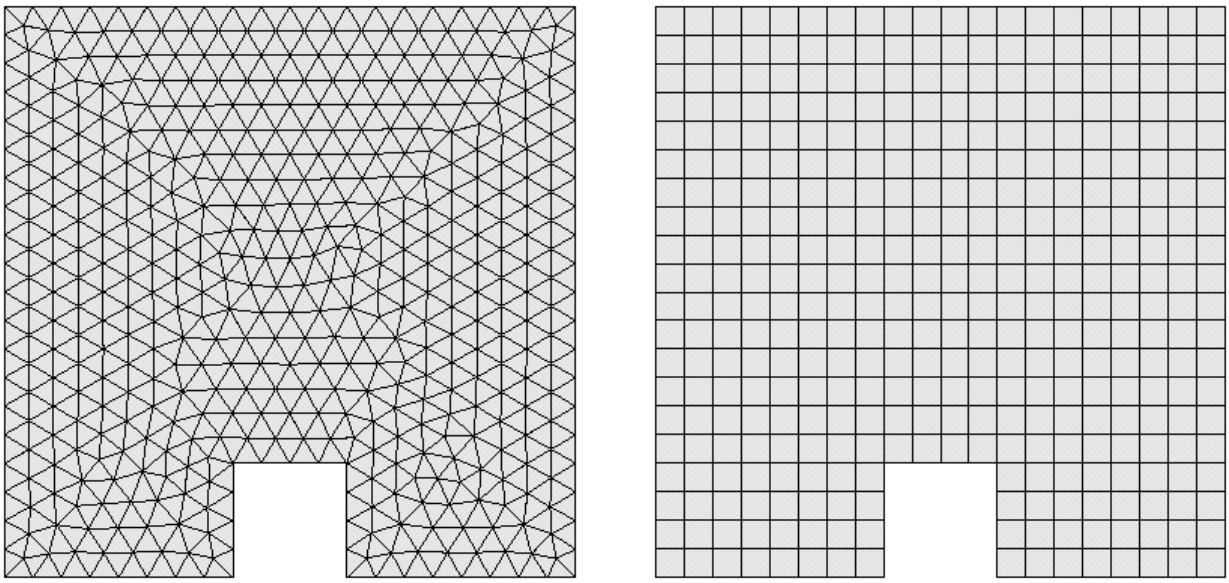


Figure 17 – Hole adjacent to the external contour (T3 and Q4).

II-8 BACKGROUND MESH

Sometimes it is not convenient to specify the target mesh sizes at the hard nodes. Non-regular variations of the sizes inside the domain can be needed but the use of many hard nodes scattered over the domain should be avoided. This is the case especially when automatic mesh adpativity is involved. The "background mesh" option is the solution in this case.
The background mesh is an auxiliary mesh used by the mesher to find the target mesh size at any point inside the domain. Hence, the user controls exactly the size chart on the entire domain.
This background mesh is represented by the connectivity matrix `background_mesh` in the data of the mesher. As always, the indices of the nodes refer to columns in the same `pos` matrix as all other connectivity matrices or vectors (such as `connectM`). The nodes of the background mesh must all have a valid associated size value in the `metrics` array.

In the following example, a regular structured background mesh is used to support a size field with a sinusoidal variation in the two directions. The domain to be meshed is a simple square regularly discretized along its boundaries[21].

[21] *For a change, we use here the `mesh_straight` overload with the parameters for the sizes at the extremities.*

```
#include "stdafx.h"

int main()
{
   const double      L  = 4.0;
   const double      h0 = 0.25;
   const double      h1 = 0.05;
   DoubleMat         pos;
   UIntVec           indices;
   UIntMat           connectE2, connectT3, BGM;
   DoubleVec         sizes;
   unsigned          N_BGM, n;
   double            w, h;

   // UNLOCK THE DLL.
   triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   // VERTICES AND LINE MESHES.
   pos.push_back (DoubleVec2(-L/2,-L/2));
   pos.push_back (DoubleVec2 (+L/2,-L/2));
   pos.push_back (DoubleVec2 (+L/2,+L/2));
   pos.push_back (DoubleVec2 (-L/2,+L/2));
   meshtools1d::mesh_straight (pos, 0, 1, h0, h0, true, indices);
   indices.pop_back();
   meshtools1d::mesh_straight (pos, 1, 2, h0, h0, true, indices);
   indices.pop_back();
   meshtools1d::mesh_straight (pos, 2, 3, h0, h0, true, indices);
   indices.pop_back();
   meshtools1d::mesh_straight (pos, 3, 0, h0, h0, true, indices);
   meshtools1d::indices_to_connectE2 (indices, connectE2);

   // THE BACKGROUND MESH.
   N_BGM = unsigned(L/h1);   // The discretization for the background mesh.
   indices.clear();
   meshtools1d::mesh_straight (pos, 0, 1, N_BGM, indices);
   indices.pop_back();
   meshtools1d::mesh_straight (pos, 1, 2, N_BGM, indices);
   indices.pop_back();
   meshtools1d::mesh_straight (pos, 2, 3, N_BGM, indices);
   indices.pop_back();
   meshtools1d::mesh_straight (pos, 3, 0, N_BGM, indices);
   meshtools2d::mesh_struct_T3 (pos, indices, N_BGM, true, BGM);

   // THE METRICS ON THE BACKGROUND MESH.
   indices.clear();
   meshtools::unique_indices (indices, BGM);
   sizes.resize (pos.cols(), 0.0);      // Null value for nodes not in BGM.
   for (unsigned i = 0; i < indices.size(); ++i)
   {
      n = indices[i];
      w = std::max(::fabs(pos(0,n)), ::fabs(pos(1,n)));
      h = ::cos(8.*M_PI*w/L) * (h0-h1)/2. + (h0+h1)/2.;
      sizes[n] = h;
   }

   // THE 2D MESH.
   triamesh::mesher               the_mesher;
   triamesh::mesher::data_type    data (pos, connectE2);
   data.background_mesh = BGM;
   data.metrics = sizes;
   the_mesher.run (data);
```



```
   // VISUALISATION.
   meshtools::medit_output ("out.mesh", data.pos, data.connectM, CM2_FACET3);

   return 0;
}   // main
```

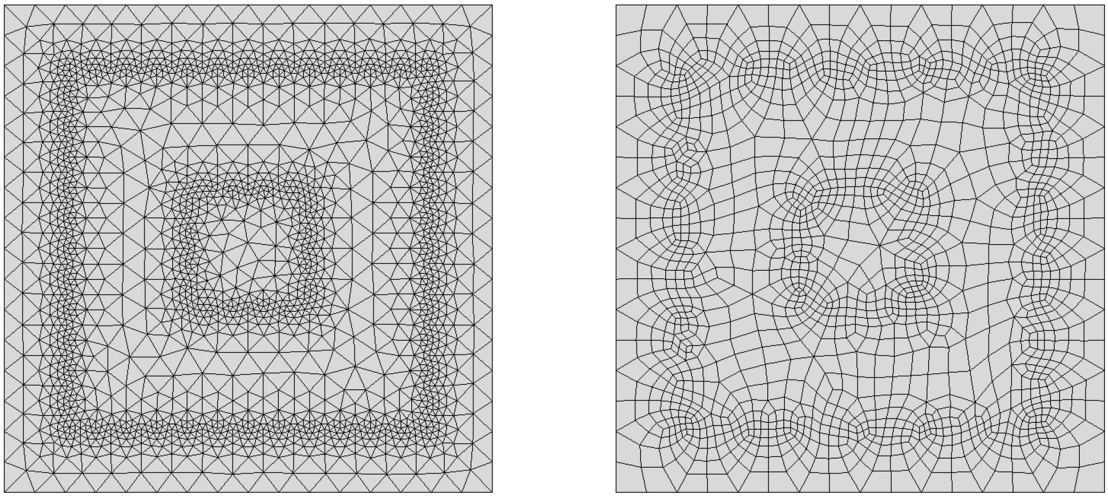


Figure 18 – Use of a background mesh to support a sizes field on the domain (T3 and all-Q4).

The background mesh is the same structured triangle mesh in both cases:

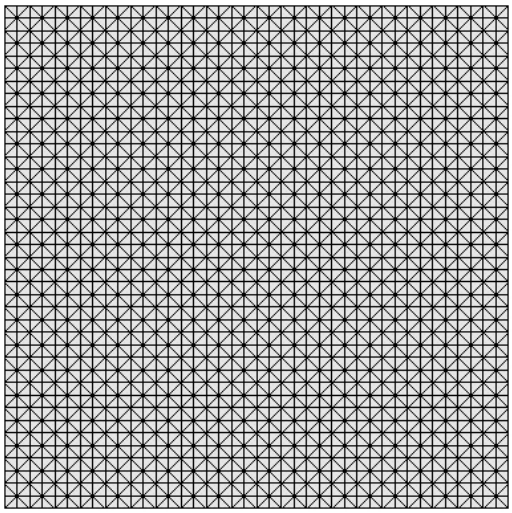


Figure 19 – The background mesh used in the previous example.

The background mesh does not need to fit exactly the domain to be meshed. It can cover only a small part of it or be partially outside of the domain. In the areas not covered by the background mesh, the default interpolation of the sizes at the hard nodes is used instead.

Here is an example where the domain is a disk and the background mesh is also a disk but with half the radius. We have set a uniform value for the sizes field on the background mesh to get a finer mesh in this area.

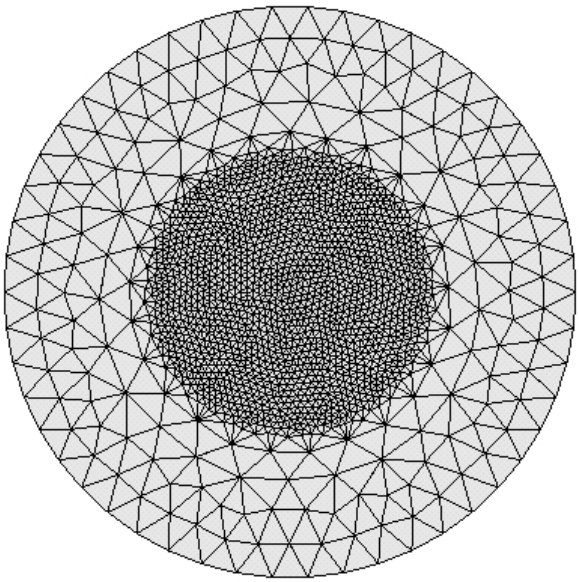


Figure 20 – Background mesh covering only a fraction of the domain.

The next step is when the boundary mesh of the domain must also be governed by a background mesh. In addition to the 2D-background mesh, we need also to discretize the boundary in order to support the sizes field on this line. Then, the real boundary mesh is generated using this 1D "background mesh" and the associated sizes. The discretization for this 1D "background mesh" must be fine enough to represent accurately the geometry of the line.
An overload of the `meshtools1d::mesh_line` function does this job[22]. Here, we discretize a full circle with 200 nodes in `indices0` and keep the associated parameters in the `U0` vector:

```
n_bgm = R / 100.;
meshtools1d::extrude_rotate (pos, 1, CR, 2*M_PI, n_bgm, indices0);
indices0.back() = indices0.front();
```

Sizes are specified on the nodes of this arc and on the 2D-background mesh as well. Then, the circle is remeshed using the initial discretization of the circle and the new sizes field:

```
vecvec::push_back (sizes, indices0, sizes0);
cm2::meshtools1d::mesh_line (pos, indices0, sizes0, true, 1,
                             UINT_MAX, 0.0, indices, new_U);
meshtools1d::indices_to_connectE2 (indices, connectE);
```

The parameters `true`, `1`, `UINT_MAX` and `0.0` stand for: force even number of edges, minimum of 1 edge, maximum of `UINT_MAX` edges along the arc and no chordal control[23].
The `indices` vector contains now the nodes of the real boundary mesh.
`new_U` contains their parameter values along the circle, but this vector is not used in the rest of the example.

[22] *Several overloads for `mesh_straight`, `mesh_spline` and `mesh_line` exist in the meshtools1d library.*
[23] *See online doc for more info on these parameters.*


```
#include "stdafx.h"

int main()
{
   const double            R   = 4.0;
   const double            h0  = 0.5;
   const double            h1  = 0.1;
   const double            sig = 0.2;
   const DoubleVec2        CR (0,0);
   const DoubleVec2        P0 (R,0);
   DoubleMat               pos;
   DoubleVec               sizes, sizes0;
   UIntVec                 indices0, indices;
   UIntMat                 connectE, connectM, BGM;
   unsigned                n_bgm, n;
   double                  x, y, w, w0, w1, w2, h;
   DoubleVec               U0, new_U;

   // UNLOCK THE DLL.
   triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   triamesh::mesher        the_mesher;

   pos.push_back (CR);
   pos.push_back (P0);

   // THE 2D BACKGROUND MESH
   n_bgm = unsigned(2.*M_PI*R / h1);   // The discretization for the BGM.
   meshtools1d::extrude_rotate (pos, 1, CR, 2*M_PI, n_bgm, indices);
   indices.back() = indices.front();   // CLose the circle.
   meshtools1d::indices_to_connectE2 (indices, connectE);
   triamesh::mesher::data_type    BGMdata (pos, connectE);
   the_mesher.mode.optim_level = 0;    // No need to have a very good BGM.
   the_mesher.run (BGMdata);
   BGMdata.extract (pos, BGM);

   // MESH THE GEOMETRIC SUPPORT OF THE BOUNDARY (1D BACKGROUND MESH).
   n_bgm = R / 100.;
   meshtools1d::extrude_rotate (pos, 1, CR, 2*M_PI, n_bgm, indices0);
   indices0.back() = indices0.front();

   // THE METRICS ON THE BACKGROUND MESHES.
   indices.clear();
   meshtools::unique_indices (indices, BGM);
   std::copy (indices0.begin(), indices0.end(), back_inserter(indices));
   sizes.resize (pos.cols(), 0.0);
   for (unsigned i = 0; i < indices.size(); ++i)
   {
      n = indices[i];
      x = pos(0,n);
      y = pos(1,n);
      w0 = ::fabs(y + 2.*x - R/2);
      w1 = ::fabs(y - x- R/2);
      w2 = ::fabs(x + R/2);
      w = std::min(w0, w1);
      w = std::min(w, w2) / sig;
      h = 1. / ((1./h0) + (1./h1) * ::exp(-w*w));
      sizes[n] = h;
   }

   // Pickup the sizes along the circle.
   sizes0.clear();
```

```
   vecvec::push_back (sizes, indices0, sizes0);

   // MESH THE EXTERNAL CONTOUR.
   indices.clear();
   connectE.clear();
   cm2::meshtools1d::mesh_line (pos, indices0, sizes0, true, 1,
                                UINT_MAX, 0.0, indices, new_U);
   meshtools1d::indices_to_connectE2 (indices, connectE);

   // THE 2D MESH.
   triamesh::mesher::data_type    data (pos, connectE);
   data.background_mesh = BGM;
   data.metrics = sizes;
   the_mesher.mode.reset();         // Reset to default values.
   the_mesher.run (data);

   // VISUALISATION.
   meshtools::medit_output ("out.mesh", data.pos, data.connectM, CM2_FACET3);

   return 0;
}   // main
```

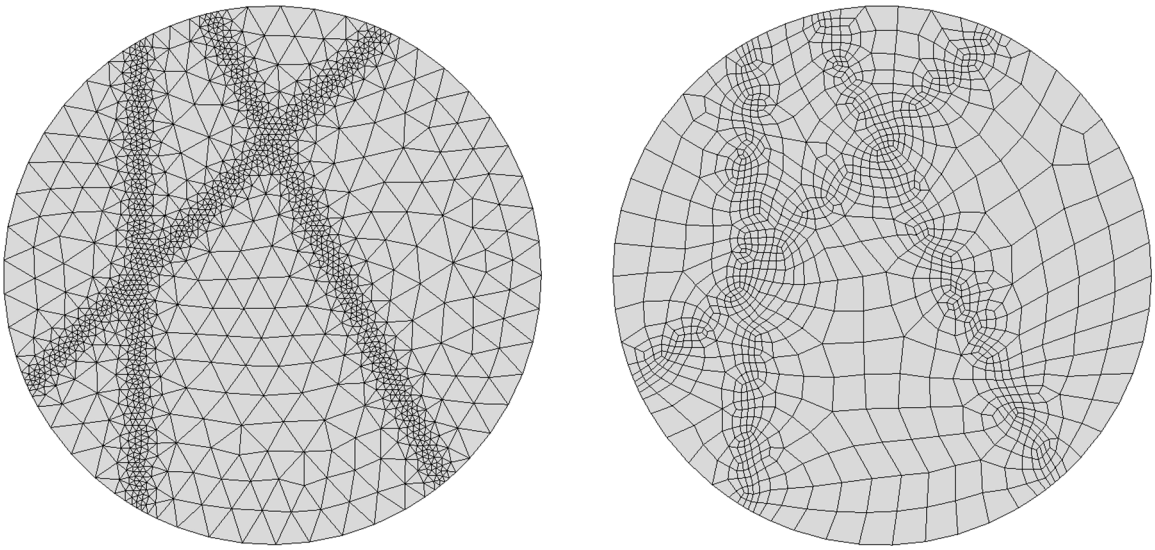


Figure 21 – Background meshes for both the boundary and the domain (T3 and all-Q4).

II-9 ANISOTROPIC MESHES

CM2 TriaMesh and CM2 QuadMesh are *isotropic* unstructured meshers, that is, they tend to produce equilateral triangles and squares. It is sometimes useful however to have elements "stretched" in some special directions. To deal with complex domains, we still need an unstructured mesher. Here come the *anisotropic* unstructured meshers CM2 TriaMesh Aniso and CM2 QuadMesh Aniso. They are almost identical to their isotropic counterparts except for the `data.metrics` array that is now a matrix. In the isotropic case, we need only a scalar at each node to define the target mesh size. Now, the target mesh size is defined by a 2x2 symmetric matrix at each node, stored column-wise in the metrics array.

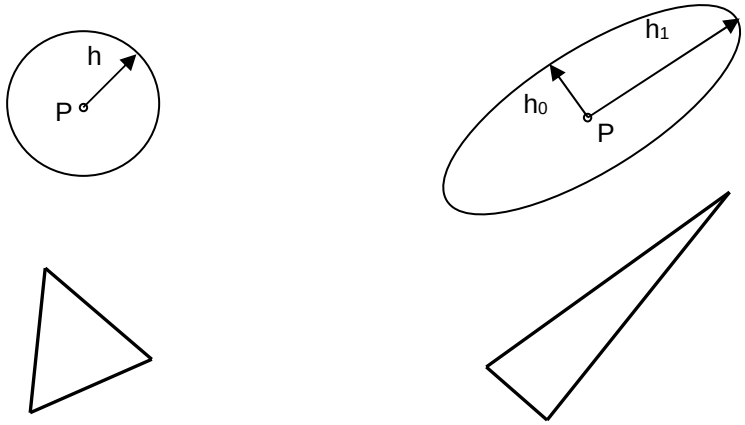


Figure 22 – A single scalar defines an isotropic metric (left).
A 2D-anisotropic metric needs two vectors (right).

$$M_j = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

with :

$a > 0$

$a\,c - b^2 > 0$

i.e. the two eigen values are > 0

$$data.metrics = \begin{bmatrix} . & . & . & a & . & . & . \\ . & . & . & b & . & . & . \\ . & . & . & c & . & . & . \end{bmatrix}$$

column #j

Figure 23 – Definition and storage of the 2-D anisotropic metrics.

Let ($\mathbf{v_0}$, $\mathbf{v_1}$) be the two orthonormal vectors along the axes of the ellipse:

$$\|\mathbf{v}_0\| = \|\mathbf{v}_1\| = 1$$
$$\langle \mathbf{v}_0, \mathbf{v}_1 \rangle = 0$$

Then, the metrics M_j writes:

$$M_j = \mathbf{B} \begin{bmatrix} \frac{1}{h_0^2} & 0 \\ 0 & \frac{1}{h_1^2} \end{bmatrix} {}^T\mathbf{B}$$
$$with:$$
$$\mathbf{B} = \begin{bmatrix} \mathbf{v}_0 & \mathbf{v}_1 \end{bmatrix}$$
$$stored\ column-wise$$

The metric equivalent to an isotropic size of *h* writes:

$$M_j = \begin{bmatrix} \frac{1}{h^2} & 0 \\ 0 & \frac{1}{h^2} \end{bmatrix}$$

A null matrix would lead to infinite sizes in both directions (infinite circle).

When the user doesn't specify a metric, the mesher uses the default one which is equivalent to the isotropic default metrics we have seen before. For each hard node, the default metric is based on the length of the adjacent edges. This leads to the same default behavior as their related isotropic counterparts. Take for instance examples II-1, II-2, II-3 or II-4 and replace:

```
triamesh::mesher        the_mesher;
```

with:

```
triamesh_aniso::mesher the_mesher;
```

and you get the same meshes[24].

To benefit from the anisotropic features, the user must fill the `metrics` array with valid anisotropic matrices (i.e. positive-definite matrices). Some functions in meshtools and meshtools1d can help in computing these matrices, as in the following example.

[24] *The anisotropic meshers are however significantly slower than their isotropic counterparts.*

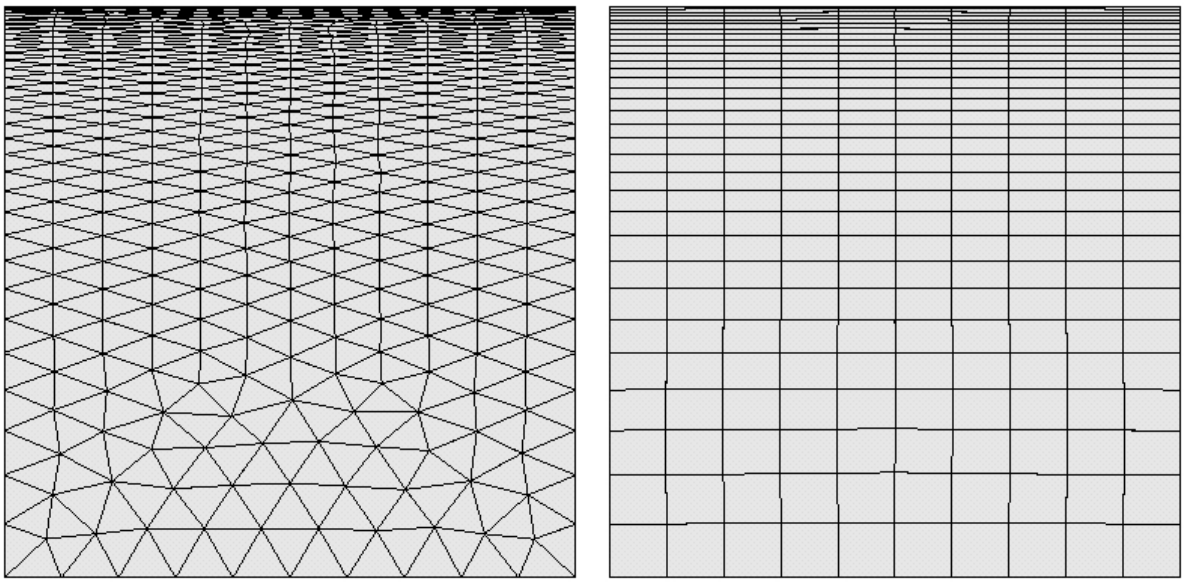


Figure 24 – Anisotropic meshes (T3 and Q4).

Here a square is meshed non-uniformly with the variant of `mesh_straight` we have already seen in the previous section[25]. This is not sufficient to get a 2-D anisotropic mesh. We need an anisotropic mesher. We need also to specify that we want a different size along the normals than along the tangents of the boundary lines (along the tangents the default sizes, i.e. mean of the edges' lengths, suit us). This is the role of `meshtools1d::metrics_gen_aniso2d`. This function takes a 1D mesh and a size along the normal and generates a set of 2-D anisotropic metrics stored in array `metrics` as depicted in Figure 23. At each node N_i, a metric $M(N_i, h_n)$ is computed. For instance, along the right vertical line we specify a constant size *hx* in the horizontal direction[26]:

```
meshtools1d::metrics_gen_aniso2d (pos, connect2, /*hn=>*/ hx, metrics);
```

[25] *We could obviously get about the same structured Q4 mesh with* `meshtools2d::mesh_struct_Q4`.

[26] *Note that the* `metrics` *parameter is not a pure out parameter. Indeed, this function does not simply overwrite the existing columns in* `metrics` *but replace them with their intersection with the newly computed metric* $M(N_i, h_n)$. *If* M_i *in column #i already exists in metrics,* M_i *is replaced by intersection* $(M_i, M(N_i, h_n))$. *Using the ellipse representation of the metrics, intersection* (M_i, M_j) *is the ellipse inscribed inside the two associated ellipses. Note also that a null metric is equivalent to an infinite circle, and that intersection (Mi, 0) = Mi*
This property of the `metrics_gen_aniso2d` *function is essential to make coherant the intersections of the generated metrics at the four sommits of the square.*

```
#include "stdafx.h"

int main()
{
   const double          L   = 10.0;
   const double          hx  = 1.0;
   const double          h0y = hx;          // Y size at bottom line.
   const double          h1y = hx / 20.;    // Y size at top line.
   const DoubleVec2      P0(0,0);
   const DoubleVec2      P1(L,0);
   const DoubleVec2      P2(L,L);
   const DoubleVec2      P3(0,L);
   DoubleMat             pos;
   UIntVec               indices;
   UIntMat               connect1, connect2, connect3, connect4, connectE;
   UIntMat               connectM;
   DoubleMat             metrics;

   // UNLOCK THE DLL.
   triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   // VERTICES
   pos.push_back (P0);
   pos.push_back (P1);
   pos.push_back (P2);
   pos.push_back (P3);

   // BOTTOM LINE
   indices.clear();
   meshtools1d::mesh_straight (pos, 0, 1, hx, hx, true, indices);
   meshtools1d::indices_to_connectE2 (indices, connect1);
   connectE.push_back (connect1);

   // RIGHT-SIDE LINE
   indices.clear();
   meshtools1d::mesh_straight (pos, 1, 2, h0y, h1y, true, indices);
   meshtools1d::indices_to_connectE2 (indices, connect2);
   connectE.push_back (connect2);

   // LEFT-SIDE LINE
   indices.clear();
   meshtools1d::mesh_straight (pos, 2, 3, hx, hx, true, indices);
   meshtools1d::indices_to_connectE2 (indices, connect3);
   connectE.push_back (connect3);

   // TOP LINE
   indices.clear();
   meshtools1d::mesh_straight (pos, 3, 0, h1y, h0y, true, indices);
   meshtools1d::indices_to_connectE2 (indices, connect4);
   connectE.push_back (connect4);

   // METRICS
   meshtools1d::metrics_gen_aniso2d (pos, connect1, /*hn=>*/ h0y, metrics);
   meshtools1d::metrics_gen_aniso2d (pos, connect2, /*hn=>*/  hx, metrics);
   meshtools1d::metrics_gen_aniso2d (pos, connect3, /*hn=>*/ h1y, metrics);
   meshtools1d::metrics_gen_aniso2d (pos, connect4, /*hn=>*/  hx, metrics);

   // 2D MESH
   triamesh_aniso::mesher               the_mesher;
   triamesh_aniso::mesher::data_type    data (pos, connectE2);
   data.metrics = metrics;
   the_mesher.run (data);
```



```
   // VISUALISATION.
   meshtools::medit_output ("out.mesh", data.pos, data.connectM, CM2_FACET3);

   return 0;
}   // main
```

As already stated, except for the `metrics` array, the anisotropic meshers have the very same options and parameters as their isotropic counterparts. They accept internal hard lines, isolated nodes, multiple domains, shared boundaries, background meshes...
The following example illustrates the internal hard line feature. We have specified a normal size on the circle much smaller than the default tangent size (again using the same `meshtools1d::metrics_gen_aniso2d` function). For the external square, nothing was specified in the `metrics` array and the mesher used its default isotropic metrics based on the length of the adjacent edges

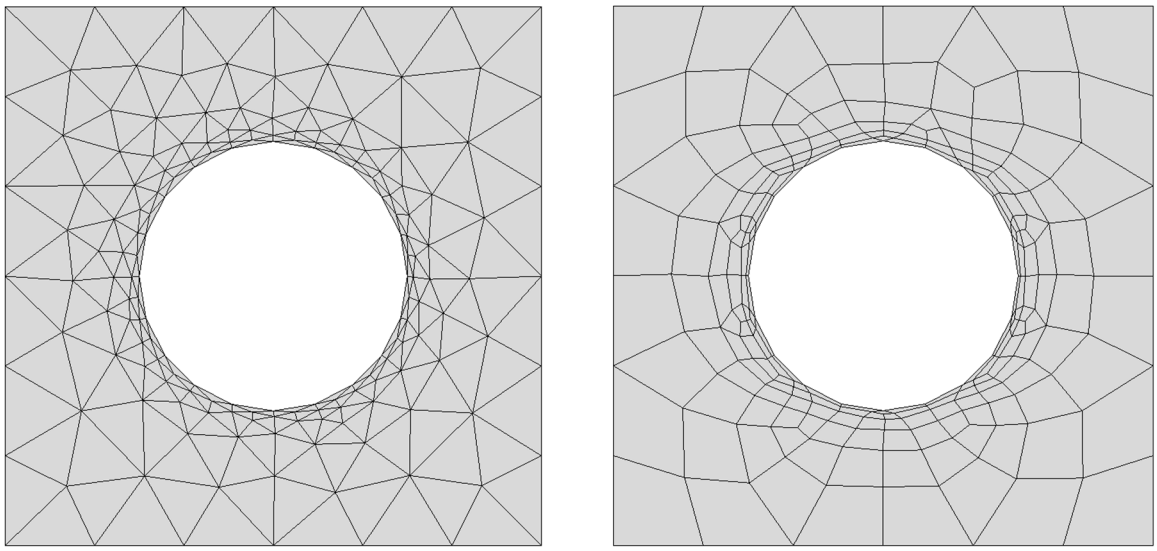


Figure 25 – 2-D anisotropic meshes (T3 and all-Q4).

The example in Figure 21 is revisited the anisotropic way. Here, we specify a small size in the directions normal to the three lines but a uniform size along the tangents. The normal sizes follow the same kind of Gaussian variation. All these metrics are specified at the nodes of the same uniform background mesh.

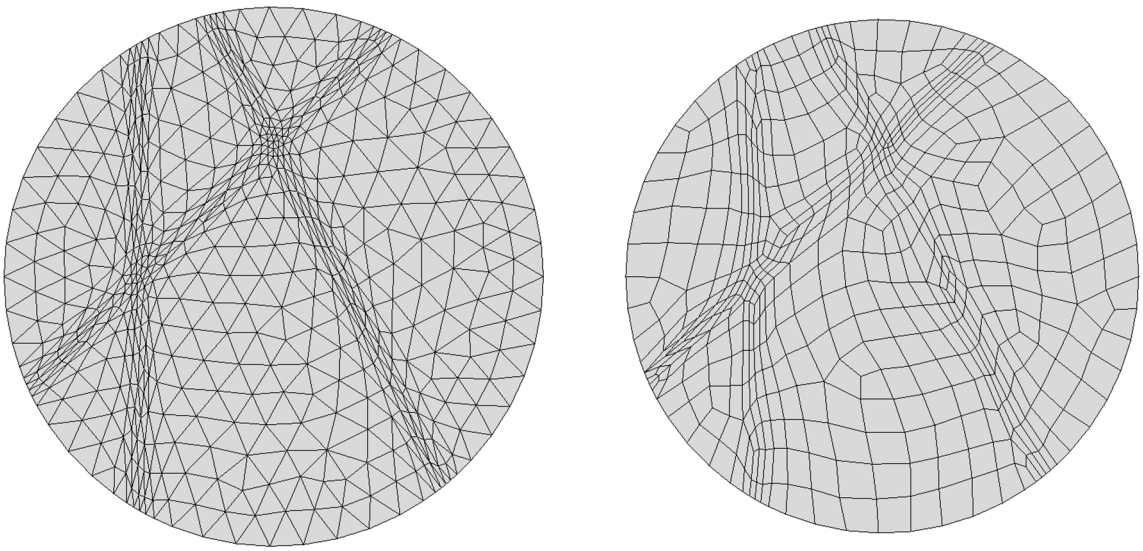


Figure 26 – 2-D anisotropic meshes (T3 and all-Q4).

II-10 3-D SURFACE MESHES (ANISO MESHERS ONLY)

The four meshers TriaMesh, QuadMesh and their anisotropic versions TriaMesh Aniso and QuadMesh Aniso are plane 2-D meshers. They generate or optimize meshes in the Z = 0 plane only. To generate meshes on 3-D parametric surfaces, CM2 MeshTools offers convenient solutions by the way of a template function that pre- and post-process the data for a 2-D anisotropic mesher (TriaMesh Aniso or QuadMesh Aniso):

```
template <class Surface, class AnisoMesher, class AuxMesher>
int
meshtools2d::mesh_surface_param
   (const Surface& S, AnisoMesher& mesher2D,
    typename AnisoMesher::data_type& data3D, AuxMesher& aux_mesher,
    double max_chordal_error, double min_h, unsigned chordal_control_type,
    unsigned high_order_type = 0, unsigned max_bgm_remeshings = 4,
    bool recompute_Qs_flag = true, bool compute_area_flag = true,
    double progress_start = 0.0, double progress_range = 1.0);
```

This function can be used as in the following code sample:

```
triamesh_aniso::mesher              the_mesher;
triamesh::mesher                    aux_mesher;
triamesh_aniso::mesher::data_type   data (pos, connectE2);
surface_type                        S(...);   // A parametric surface.
meshtools2d::mesh_surface_param (S, the_mesher, data,
                                 aux_mesher, -0.05, 0.0, 4);
data.extract (pos, connectM);
```

The class `Surface` for parameter `S` is a concept of parametric surface with members:

```
int get_3D_coordinates (const DoubleMat& pos2D, DoubleMat& pos3D) const;
int get_2D_coordinates (const DoubleMat& pos3D, const UIntVec& nodeIDs,
                        DoubleMat& pos2D) const;
int get_local_bases (const DoubleMat& pos2D, DoubleMat& local_B) const;
int get_local_curvatures (const DoubleMat& pos2D, DoubleMat& local_H) const;
```

The **`Surface::get_3D_coordinates`** member should compute the 3-D coordinates of a set of 2-D points located on the reference plane. The 3-D coordinates of the point in column #j of `pos2D` must be returned in column #j of `pos3D`.
This function should return zero when successful.

The **`Surface::get_2D_coordinates`** member is the reciprocal function of the previous one[27]. It should give the coordinates in the 2-D reference plane of a set of 3-D points. The reference coordinates of the point in column #j of `pos3D` must be returned in column #j of `pos2D`.
This function should return zero when successful
`nodeIDs` is an auxiliary vector that can be helpful for an effective implementation. It contains the global indices of the nodes for which the 2-D coordinates are required. These are the indices in the the global matrix `data3D.pos`: `nodeIDs[j]` is the node ID (i.e. column in `data.pos`) for the coordinates in

[27] *For parametric surfaces such as Bezier surfaces or NURB surfaces, the computation of reference coordinates often involves a non-linear search. However, this function is called only for the nodes on the boundary mesh and for the isolated nodes (i.e. the hard nodes only). It is not called for the new nodes generated inside the surface by the mesher.*

column `j` of `pos3D`. This array can be used for fast 2-D coordinates retrieval if these coordinates have been computed before.

The **`Surface::get_local_bases`** member should compute the two tangents $B_u = \frac{\partial P}{\partial u}$ and $B_v = \frac{\partial P}{\partial v}$ on the surface at a set of points given by their reference coordinates.
These tangents must *not* be normalized. They are the mere derivatives of the surface with respect to two reference parameters. The two tangents at the point in column #j of `pos2D` must be returned in column #j of `local_B` (dimension 6xN). The first three values are for the first tangent (with respect to the first reference coordinate), then the next three are for the second tangent[28].
This function should return zero when successful.

The **`Surface::get_local_curvatures`** function may compute the curvatures of the surface at a set of points given by their reference coordinates (optional).
The curvatures H are 2x2 symmetric matrices defined as:

- $H_{uu} = \left\langle \frac{\partial^2 P}{\partial u^2}, N \right\rangle$

  Dot product between the derivative of B_u (first local tangent) with respect to *u*, and the normal N to the surface.
- $H_{uv} = \left\langle \frac{\partial^2 P}{\partial u \partial v}, N \right\rangle$

  Dot product between the derivative of B_u (first local tangent) with respect to *v*, or derivative of B_v (second local tangent) with respect to *u*, and the normal N to the surface.
- $H_{vv} = \left\langle \frac{\partial^2 P}{\partial v^2}, N \right\rangle$

  Dot product between the derivative of B_v (second local tangent) with respect to *v*, and the normal N to the surface.

These three values must be stored column-wise in matrix `local_H`: H_{uu} on row 0, H_{uv} on row 1 and H_{vv} on row 2.
This function should return zero when successful and a negative value (-1 for instance) when failed.
You can leave the implementation of this member empty (returning -1 for instance). In this case approximative curvatures computed from variations of the tangents will be used instead.

The template class `AnisoMesher` is a concept of triangle anisometric mesher with function:

```
void run (typename AnisoMesher::data_type& data) const;
```

The `mesh_surface_param` function is designed to work with one of the 2-D anisotropic meshers CM2 TriaMesh Aniso or CM2 QuadMesh Aniso.

The `data3D` parameter is the structure gathering all the input and output data, just like for any other unstructured mesher of CM2 MeshTools. The type of `data3D` is either `triamesh_aniso::mesher::data_type` or `quadmesh_aniso::mesher::data_type`

[28] This function should normally return in `local_B` only valid bases made of two non-null and non-colinear vectors. When the surface exhibits some singularities, the user can "correct" the deficient bases. As far as the mesher is concerned, the exactness of these tangents with respect to the true surface is not critical. More precisely, the tangent bases are used by the template function as transformation matrices to compute the target anisotropic 2-D `metrics` array. The template function checks for deficient aniso metrics (derived from deficient local bases) and replace them with a default one.

depending on the type of anisotropic mesher used. The point is that the `pos` matrix is now a 3-D coordinates matrix and the `metrics` array contains 3-D anisotropic metrics (dimensions 6xNODS).

3D-anisotropic metrics are defined as below:

$$M_j = \begin{bmatrix} a & b & d \\ b & c & e \\ d & e & f \end{bmatrix}$$

$$with:$$

$$a > 0$$

$$a\,c - b^2 > 0$$

$$Det(M_j) > 0$$

$$i.e.\ the\ three\ eigen\ values\ are > 0$$

$$data3D.metrics = \begin{bmatrix} . & . & . & a & . & . & . \\ . & . & . & b & . & . & . \\ . & . & . & c & . & . & . \\ . & . & . & d & . & . & . \\ . & . & . & e & . & . & . \\ . & . & . & f & . & . & . \end{bmatrix}$$

column #j

Figure 27 – Definition and storage of the 3-D anisotropic metrics.

Let ($\mathbf{v_0}$, $\mathbf{v_1}$, $\mathbf{v_2}$) be the three orthonormal vectors along the axes of the ellipsoid:

$$\|\mathbf{v}_0\| = \|\mathbf{v_1}\| = \|\mathbf{v_2}\| = 1$$

$$\langle \mathbf{v}_0, \mathbf{v}_1 \rangle = 0$$

$$\langle \mathbf{v}_0, \mathbf{v_2} \rangle = 0$$

$$\langle \mathbf{v_1}, \mathbf{v_2} \rangle = 0$$

$$\langle \mathbf{v}_0 \times \mathbf{v_1}, \mathbf{v_2} \rangle = 1$$

Then, the metrics M_j writes:

$$M_j = \mathbf{B} \begin{bmatrix} \frac{1}{h_0^2} & 0 & 0 \\ 0 & \frac{1}{h_1^2} & 0 \\ 0 & 0 & \frac{1}{h_2^2} \end{bmatrix}^T \mathbf{B}$$

$$with:$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{v}_0 & \mathbf{v}_1 & \mathbf{v}_2 \end{bmatrix}$$

$$stored\ column-wise$$

The 3-D metric equivalent to an isotropic size of *h* writes:

$$M_j = \begin{bmatrix} \frac{1}{h^2} & 0 & 0 \\ 0 & \frac{1}{h^2} & 0 \\ 0 & 0 & \frac{1}{h^2} \end{bmatrix}$$

A null matrix would lead to infinite sizes in the three directions (infinite sphere).

The two parameters `max_chordal_error` and `chordal_control_type` are used to limit the chordal error between the mesh and the parametric surface.
We don't use them in this tutorial (set to 0). Please refer to the API documentation for more information on them.

This first example illustrates the use of the anisotropic mesh as the intermediate mesh. Here, the parametric surface to be meshed is plane but its boundaries are curved (sinusoidal). The parameters' range is the unit square [0 1] x [0 1].

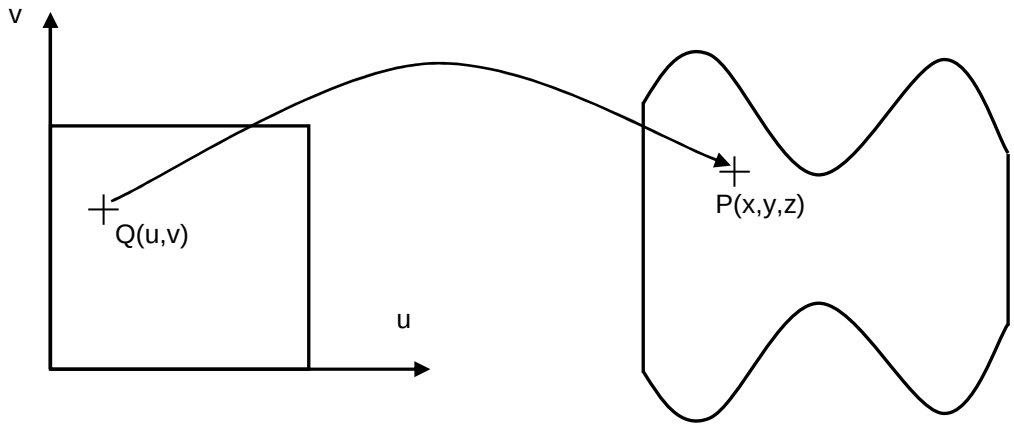


Figure 28 – Mapping between the reference space and the surface.

The source of this example is as follow:


```
#include "stdafx.h"


/*The Surface class implements the functions needed by mesh_surface_param*/
struct surface
{
// Constructor (parameters to define the surface should be passed here).
surface (double Lx, double Ly, double a = 0.5)
   : _Lx(Lx), _Ly(Ly), _a(a) { }

// Compute the 3D coordinates.
int get_3D_coordinates (const DoubleMat& pos2D, DoubleMat& pos3D) const
{
   const unsigned    NODS = pos2D.cols();
   double            u, v, x, y, z;

   if (pos2D.rows() != 2) return -1;   // Error.

   if ((pos3D.rows() != 3) || (pos3D.cols() < NODS))
      pos3D.resize (3, NODS);

   for (unsigned j = 0; j < NODS; ++j)
   {
      u = pos2D(0,j);
      v = pos2D(1,j);

      x = u * _Lx;
      y = v * _Ly * (1. + _a * ::sin(x));
      z = 0.0;

      pos3D(0,j) = x;
      pos3D(1,j) = y;
      pos3D(2,j) = z;
   }
   return 0;      // OK.
}

// Compute the reference coordinates (UV). nodeIDs not used.
int get_2D_coordinates (const DoubleMat& pos3D, const UIntVec& nodeIDs,
                        DoubleMat& pos2D) const
{
   const unsigned    NODS = pos3D.cols();
   double            u, v, x, y;

   if (pos3D.rows() != 3) return -1;   // Error.

   if ((pos2D.rows() != 2) || (pos2D.cols() < NODS))
      pos2D.resize (2, NODS);

   for (unsigned j = 0; j < NODS; ++j)
   {
      x = pos3D(0,j);
      y = pos3D(1,j);

      u = x / _Lx;
      v = y / (_Ly * (1. + _a * ::sin(x)));

      pos2D(0,j) = u;
      pos2D(1,j) = v;
   }
   return 0;      // OK.
}
```

```
/// Compute the local tangents.
int get_local_bases (const DoubleMat& pos2D, DoubleMat& B) const
{
   const unsigned    NODS = pos2D.cols();
   double            u, v, x;

   if (pos2D.rows() < 2) return -1;   // Error.

   if ((B.rows() != 6) || (B.cols() < NODS))
      B.resize (6, NODS);

   for (unsigned j = 0; j < NODS; ++j)
   {
      u = pos2D(0,j);
      v = pos2D(1,j);
      x = u * _Lx;

      B(0,j) = _Lx;
      B(1,j) = v * _Ly * _a * _Lx * ::cos(x);
      B(2,j) = 0.0;
      B(3,j) = 0.0;
      B(4,j) = _Ly * (1. + _a * ::sin(x));
      B(5,j) = 0.0;
   }
   return 0;      // OK.
}

/// Compute the local curvatures.
int get_local_curvatures (const DoubleMat& pos2D, DoubleMat& H) const
{
   const unsigned    NODS = pos2D.cols();

   if (pos2D.rows() < 2) return -1;   // Error.

   if ((H.rows() != 3) || (H.cols() < NODS))
      H.resize (3, NODS);

   H = 0.0;     // Null curvatures here (the surface is plane).
   return 0;
}

/// Data members.
double      _Lx, _Ly, _a;

};   // surface



///
int main()
{
   const DoubleVec2        P0 (0.0, -0.5);
   const DoubleVec2        P1 (1.0, -0.5);
   const DoubleVec2        P2 (1.0, +0.5);
   const DoubleVec2        P3 (0.0, +0.5);
   DoubleMat               pos;
   UIntVec                 indices0, indices1, indices2, indices3, indices;
   DoubleVec               U0, U1, U2, U3, new_U;
   UIntMat                 connectE2, connectM;
   DoubleVec               sizes;
   const double            Lx = 10.0;
```



```
   const double            Ly = 6.0;
   const double            h0 = 0.25;
   surface                 S (Lx, Ly, 0.5);  // The parametric surface
   char                    filename[64];

   // UNLOCK THE DLLs.
   triamesh_aniso::registration ("Licensed to SMART Inc.", "B657DA67QZ01");
   triamesh::registration ("Licensed to SMART Inc.", "F53EA108BCWX");

   pos.push_back (P0);
   pos.push_back (P1);
   pos.push_back (P2);
   pos.push_back (P3);


   // THE GEOMETRIC SUPPORT OF THE EXTERNAL CONTOUR.
   sizes.resize (4, 1.0 / 100);
   meshtools1d::mesh_straight (pos, 0, 1, UIntVec(), DoubleVec(), sizes,
                               true, indices0, U0);
   meshtools1d::mesh_straight (pos, 1, 2, UIntVec(), DoubleVec(), sizes,
                               true, indices1, U1);
   meshtools1d::mesh_straight (pos, 2, 3, UIntVec(), DoubleVec(), sizes,
                               true, indices2, U2);
   meshtools1d::mesh_straight (pos, 3, 0, UIntVec(), DoubleVec(), sizes,
                               true, indices3, U3);
   S.get_3D_coordinates (pos, pos);   // Map the contour onto the surface.


   // MESH THE EXTERNAL CONTOUR BASED ON THE PREVIOUS GEOMETRIC SUPPORT.
   sizes.clear();
   sizes.resize (pos.cols(), h0);
   meshtools1d::mesh_straight (pos, 0, 1, indices0, U0, sizes,
                               true, indices, new_U);
   indices.pop_back(); new_U.pop_back();
   meshtools1d::mesh_straight (pos, 1, 2, indices1, U1, sizes,
                               true, indices, new_U);
   indices.pop_back(); new_U.pop_back();
   meshtools1d::mesh_straight (pos, 2, 3, indices2, U2, sizes,
                               true, indices, new_U);
   indices.pop_back(); new_U.pop_back();
   meshtools1d::mesh_straight (pos, 3, 0, indices3, U3, sizes,
                               true, indices, new_U);
   meshtools1d::indices_to_connectE2 (indices, connectE2);


   // SURFACIC MESH.
   triamesh_aniso::mesher              the_mesher;
   triamesh_aniso::mesher              aux_mesher;
   triamesh_aniso::mesher::data_type   data (pos, connectE2);
   meshtools2d::mesh_surface_param (S, the_mesher, data,aux_mesher,0.,0.,0);
   data.extract (pos, connectM);
   data.print_info (&display_hdl);

   // VISUALISATION.
   meshtools::medit_output ("out.mesh", data.pos, data.connectM, CM2_FACET3);

   return 0;
}   // main
```

We present below the intermediate anisotropic meshes on the reference space (normally not shown) and the final meshes on the parametric surface.

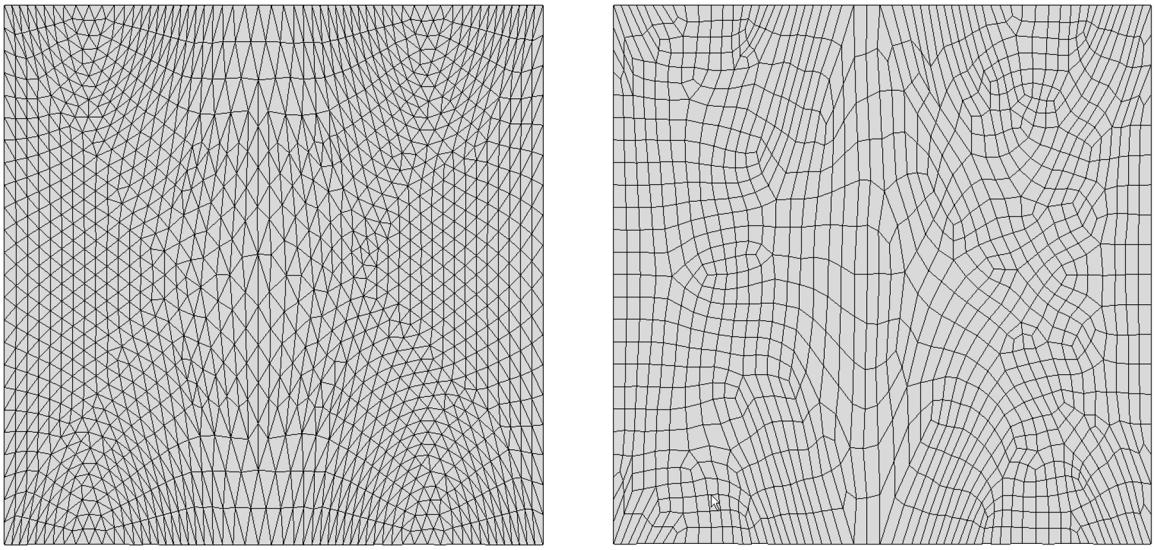


Figure 29 – 2-D anisotropic meshes in the reference space (UV).

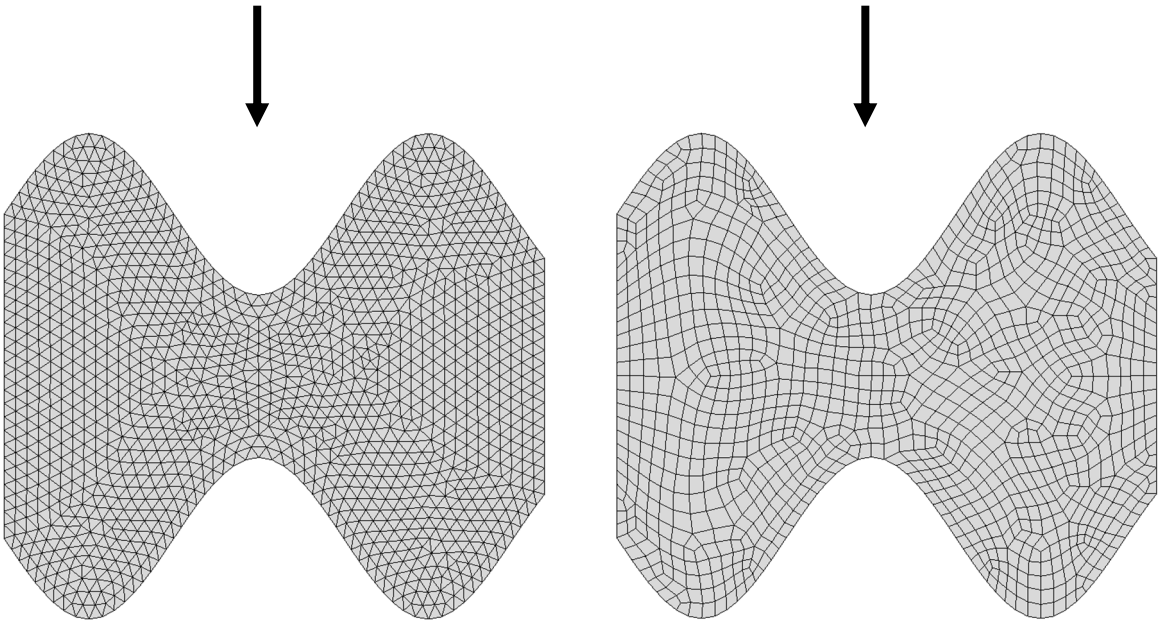


Figure 30 – Surface meshes (T3 and all-Q4) obtained via an anisotropic mesh in the reference space.

The next example is a true 3-D parametric surface (double sinusoidal). We present only the source code for the surface class.


```
/*The Surface class implements the functions needed by mesh_surface_param*/
// Constructor.
surface (double L, double H) : _L(L), _H(H) { }

// Compute the 3D coordinates.
int get_3D_coordinates (const DoubleMat& pos2D, DoubleMat& pos3D) const
{
   const unsigned    NODS = pos2D.cols();
   double            u, v, x, y, z;

   if (pos2D.rows() < 2) return -1;   // Error.

   if ((pos3D.rows() != 3) || (pos3D.cols() < NODS))
      pos3D.resize (3, NODS);

   for (unsigned j = 0; j < NODS; ++j)
   {
      u = pos2D(0,j);
      v = pos2D(1,j);

      x = _L * u;
      y = _L * v;
      z = _H * ::cos(x) * ::cos(y);

      pos3D(0,j) = x;
      pos3D(1,j) = y ;
      pos3D(2,j) = z;
   }
   return 0;      // OK.
}

// Compute the reference coordinates (UV). nodeIDs not used.
int get_2D_coordinates (const DoubleMat& pos3D, const UIntVec& nodeIDs,
                        DoubleMat& pos2D) const
{
   const unsigned    NODS = pos3D.cols();
   double            u, v, x, y;

   if (pos3D.rows() < 2) return -1;   // Error.

   if ((pos2D.rows() != 2) || (pos2D.cols() < NODS))
      pos2D.resize (2, NODS);

   for (unsigned j = 0; j < NODS; ++j)
   {
      x = pos3D(0,j);
      y = pos3D(1,j);

      u = x / _L;
      v = y / _L;

      pos2D(0,j) = u;
      pos2D(1,j) = v;
   }
   return 0;      // OK.
}

/// Compute the local tangents.
int get_local_bases (const DoubleMat& pos2D, DoubleMat& B) const
{
   const unsigned    NODS = pos2D.cols();
   double            u, v, x, y;
```

```
   if (pos2D.rows() < 2) return -1;   // Error.

   if ((B.rows() != 6) || (B.cols() < NODS))
      B.resize (6, NODS);

   for (unsigned j = 0; j < NODS; ++j)
   {
      u = pos2D(0,j);
      v = pos2D(1,j);

      x = _L * u;
      y = _L * v;

      B(0,j) = _L;
      B(1,j) = 0.0;
      B(2,j) = -_H * _L * ::sin(x) * ::cos(y);
      B(3,j) = 0.0;
      B(4,j) = _L;
      B(5,j) = -_H * _L * ::cos(x) * ::sin(y);
   }
   return 0;      // OK.
}

/// Compute the local curvatures.
int get_local_curvatures (const DoubleMat& pos2D, DoubleMat& H) const
{
   const unsigned    NODS = pos2D.cols();
   double            u, v, x, y;

   if (pos2D.rows() < 2) return -1;   // Error.

   if ((H.rows() != 3) || (H.cols() < NODS))
      H.resize (3, NODS);

   for (unsigned j = 0; j < NODS; ++j)
   {
      u = pos2D(0,j);
      v = pos2D(1,j);

      x = _L * u;
      y = _L * v;

      sx = ::sin(x);  cx = ::cos(x);
      sy = ::sin(y);  cy = ::cos(y);
      s = _H *_L*_L / ::sqrt(1. + _H*_H * (sx*sx*cy*cy + cx*cx*sy*sy));
      H(0,j) = - cx*cy * s;
      H(1,j) = + sx*sy * s;
      H(2,j) = - cx*cy * s;
   }
   return 0;
}


// Data members.
double      _L, _H;
};    // surface.
```

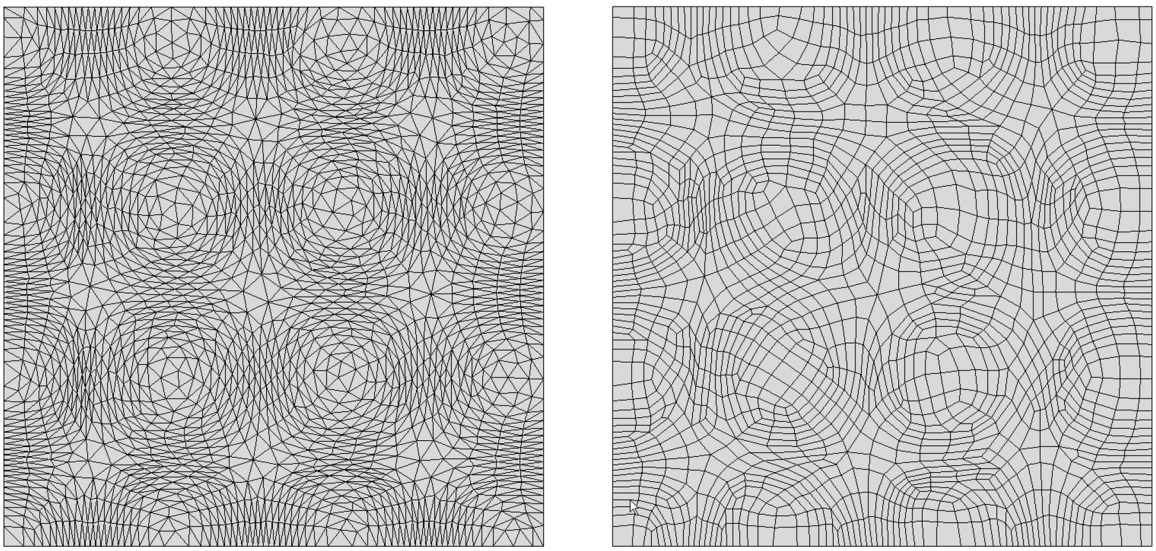



Figure 31 – 2-D anisotropic meshes in the reference space (UV).

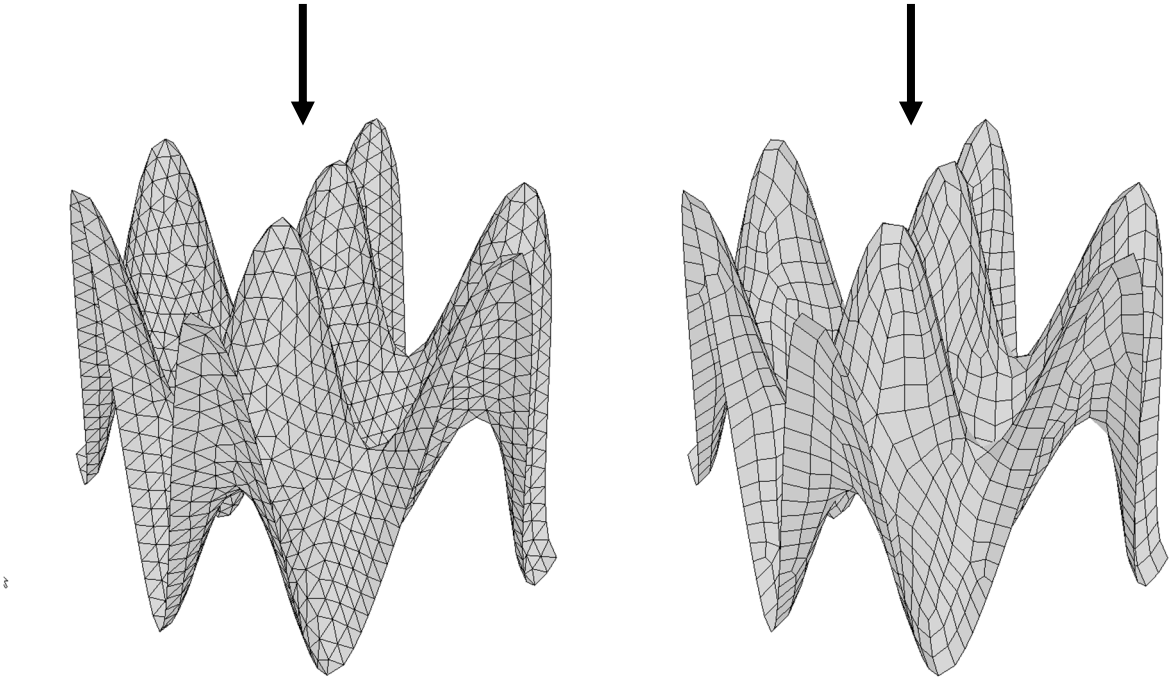


Figure 32 – 3-D surface meshes (T3 and all Q4).

Notes:

- This solution for 3-D surface meshing can be used only when a “mathematical” representation of the surface is available (through a CAD kernel for instance). This method is implemented in **CM2 SurfMesher T3** and **CM2 SurfMesher Q4** (experimental CM2 components using the OpenCascade® kernel).
- When there is only a discrete representation of the surface available (such as a tessellated surface), a different method can be used: 3-D patch remeshing. This method is implemented in CM2 SurfRemesher T3 and CM2 SurfRemesher Q4, two other components of the CM2 MeshTools library. For more information, please read "3-D Surface Remeshers – Tutorial and User's Manual".
- A similar template function (`meshtools1d::mesh_curve_param`) is available for parametric curve meshing.

III – USER'S MANUAL

III-1 MESH GENERATORS' DATA

All data for a run of the mesher are gathered into a single structure of type `data_type`:

```
void triamesh::mesher::run (triamesh::mesher::data_type& data);
void quadmesh::mesher::run (quadmesh::mesher::data_type& data);
void triamesh_aniso::mesher::run (triamesh_aniso::mesher::data_type& data);
void quadmesh_aniso::mesher::run (quadmesh_aniso::mesher::data_type& data);
```

Some of the most important fields of these structures have already been seen in the previous chapter. This part recalls these points and details all the others.

Coordinates of the points

Matrix `pos` of dimensions 2xN (IN-OUT).

This matrix stores the coordinates of *all* points. The coordinates are stored column-wise. The column index is the index of the node (zero-based, i.e. from 0 to N-1). The *X* coordinates are in the first row and the *Y* coordinates in the second row.
Upon exit, the coordinates of the newly generated nodes are appended to the back of the matrix as new columns. The initial columns are left unchanged.

Hard edges

Matrix `connectB` of dimensions 2xB (IN-OUT).

This matrix stores the connectivity of the edges of the boundary mesh and, if any, the edges of the constrained internal lines. These edges are also called *hard edges*. The node IDs of the edges are stored column-wise. `connectB(i,j)` is the i^{th} (0 or 1) node of the j^{th} edge.
This 1D mesh must comply with three conditions:

- It must be valid - no edge cuts, no degenerated edges[29].
- It must contain at least one closed external boundary.
- If holes or several external boundaries are present, all the edges of the external contour(s) must be oriented the same way - for instance, all counter-clockwise. Edges in the contour of the holes must be oriented the opposite way - for instance clockwise.

This matrix can be left empty upon entry when the mesher is used in the optimizer mode or in the convex hull mode (cf. III-3 Options of the mesh generators, Basic operating mode). The user can let the mesher to recalculate this boundary mesh - totally or partially - by setting the flag `reset_external_boundaries_flag = true` (see § III-3). In this case, the `connectB` matrix is also an output data.
The ordering of the edges in the matrix, i.e. ordering of the columns, is irrelevant[30]. Edges of holes can for instance be mixed with edges of the external boundary.
Edges can also be oriented both ways. For instance, some edges of the contour of a hole (so-called negative contour) can be shared with edges of a positive contour (same nodes but opposite orientation). See example II-7.

Isolated (embedded) hard nodes

Vector `isolated_nodes` (IN).

---

[29] Condition #1 can be partially cancelled with flag `strict_constraints_flag = false`. With this flag down, cuts between boundary edges are allowed as well as isolated nodes inside an edge (but a warning is echoed). See § III-3.
[30] However, the ordering is taken into account when boundaries are intersecting and mode is non-strict enforcement. In this case, only the lowest intersecting edges are kept..

This vector stores the index of the points in the `pos` matrix that the user wants to be present in the final mesh[31] in top of the hard edges nodes. These points must be geometrically distinct and not located on a hard edge[32].
These nodes together with the nodes of the hard edges make the set of the so-called *hard nodes*.

Repulsive points

Vector `repulsive_points` (IN).

This vector contains the index of the points stored in the `pos` matrix that will define areas of node repulsion in the final mesh. These repulsive points must be associated with valid metrics (mesh sizes) to be properly taken into account. Because of the mesh sizes at the neighboring hard nodes, the disks are mere repulsion disks, not exclusion disks. The closer to the repulsive points, the less the probability to find a generated node.

Background mesh

Matrix `background_mesh` (IN) of dimensions 3xMb.

This is the connectivity matrix of an auxiliary mesh used to interpolate the metric field. Like the other connectivity matrices (`connectB`, `connectM`), the indices refer to columns in the `pos` coordinates matrix. The metrics at the nodes of this background mesh are given in the `metrics` array. All nodes of the background mesh must have a valid metric in this array (i.e. a positive value for the isotropic meshers, a matrix with two positive eigen values in the anisotropic cases). This mesh must not contain any degenerated or badly oriented elements.
It can be used to control precisely the size of the elements on the entire domain, not only from the hard nodes of the boundaries. This is very useful for instance in a mesh adaptivity scheme in FEM analysis. The mesh at step N+1 (`connectM`) is generated using the mesh at step N as the background mesh.
The background mesh does not need to cover the entire domain. You can define a background mesh only on a part of the domain and leave the default metric interpolation outside.
Note also that the background mesh is always a triangle mesh, even for the quad meshers[33].
If left empty, a default background mesh is used instead[34].

Metric field

Vector or matrix `metrics` (OUT or IN-OUT).
Vector for the isotropic meshers, matrix with 3 rows for the anisotropic meshers.

Upon entry, the user can specify a target mesh size on each hard node or each node of the background mesh. If the value for a node is zero - or negative or not present -, the automatically computed value will be used instead[35]. The 2-D mesh is generated to fit best the metric field all over the domain.
For better results, it is recommended to specify a size value (or metric matrix) only on isolated nodes and leave the automatic values, i.e. set to zero, on the nodes of the hard edges.
Another solution is to use a background mesh but the user has to specify the target metric at each of its nodes.
Note that a steep gradient in the metric field renders the task of the mesher more difficult and surely affects the quality of the elements.

[31] However, if such an isolated point is located outside the domain (or inside a hole), it will not be present in the final mesh. Its coordinates are left unchanged in the `pos` matrix.
[32] Except if flag `strict_constraints_flag = false` (see § III-3).
[33] The `meshtools2d::split_Q4_into_T3` can be used to convert a quad mesh into a triangle mesh.
[34] This is the "front" mesh (triangulation mesh of the hard nodes only) when the `use_default_background_mesh_flag` flag in the operating mode is set to true. If it is set to false, the background mesh is simply the current mesh in the meshing process. It is recommended to leave this flag to false.
[35] For a node of a boundary or internal line, the computed size is the average length of the coincident edges. For an isolated node, the computed size is based on the size value of the nearest nodes.

Elements

Matrix `connectM` (OUT or IN-OUT) of the connectivity of the elements (column oriented).

The node IDs of the elements are stored column-wise: `connectM(i,j)` is the i^{th} node of the j^{th} element. They refer to columns in the coordinates matrix `pos`. The local numbering of the nodes is shown Figure 33.
For the triangle meshers, the dimensions of this matrix are always 3xNEFS.
For the quadrangle meshers, the dimensions are either 4xNEFS (in `MESH_MODE` and in `REGULARIZE_MODE`) or 3xNEFS (in `CONVEX_HULL_MODE` because only triangles are generated in this case). In `MESH_MODE` and in `REGULARIZE_MODE`, the leading part of this matrix (from columns 0 to `nefs_Q4`-1) is the connectivity of the quadrangle elements. The tailing part of this matrix (from columns `nefs_Q4` to `nefs-1`) is the connectivity of the triangles elements. The fourth node ID in this part of the matrix is always `CM2_NONE` (i.e. `unsigned(-1)`).
This matrix can be non-empty upon entry when the mesher is used as an optimizer of an already existing mesh (`REGULARIZE_MODE`). Otherwise, it is always an output matrix.
The ordering of the elements in this matrix is irrelevant.

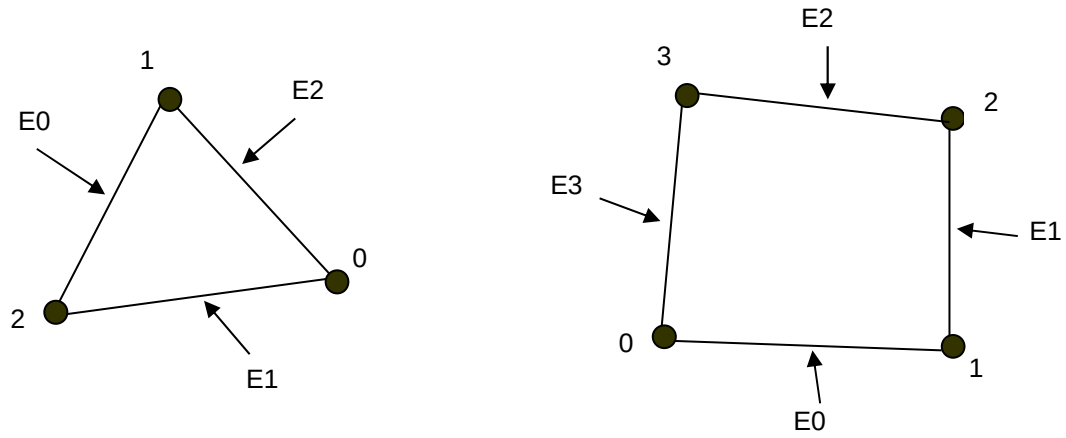


Figure 33 – Nodes and edges local numbering in triangles and quads.

The elements (triangles or quads) are always oriented counter-clockwise - normal towards positive *Z* - whatever the orientation of the edges of the external contour.

Unenforced entities

Vectors `unenforced_boundary_IDs` and `unenforced_node_IDs` (IN and OUT).

Upon exit, these two arrays store the IDs of the entities that could not be enforced (because of intersections between hard entities or hard entities located outside the domain).
In strict-constraint mode (see § III-3), an error is raised when at least one such hard entity cannot be enforced.

Pathological boundaries

Vector `pathological_boundary_IDs` (OUT).

This vector gives the column IDs in the `connectB` matrix of the edges that intersect other hard edge(s)/node(s).
`pathological_boundary_IDs` contains all the intersecting edges. In non-strict mode (`strict_constraints_flag = false`), some of them will be present in the final mesh (the first in `connectB`), some will be missing (the highest in `connectB`).
In strict-constraints mode (`strict_constraints_flag = true`), the generator stops with an error when this vector is not empty.

Elements' color

Vector `colors` (OUT or IN-OUT) of size equal to M (same size as the number of columns in the `connectM` matrix).

For each element, this vector gives the index, the so-called *color*, of the subdomain[36] in which it belongs. Upon exit, the size of this vector equals to the number of output elements, i.e. number of columns in matrix `connectM`. The color values start at one for the most external subdomain and are incremented each time an internal closed boundary is encountered from the exterior towards the interior. If several subdomains have the same "level" with respect to the outer boundary one cannot assume any order between the colors of these subdomains.
In the optimizing mode, the user can input an initial color chart - with size equals to the number of input elements. This will be used to affect a color to the elements created by the optimizer if needed[37].

Neighbors

Matrix `neighbors` (OUT) of dimensions 3xM for the triangles and 4xM for the quads.

This matrix gives, for each element in the final mesh, the indices of the three or four neighboring elements (-1 if none).
`neighbors(i,j)` is the neighbor of the j^{th} element sharing the i^{th} edge. See Figure 33 for the local numbering of the edges.

Ancestors

Vector `ancestors` (OUT) of size N, i.e. the number of columns in the `pos` matrix.

This vector gives, for each point, the index of one of the elements in which it belongs (-1 if none, i.e. if the point is not in the final mesh).
Together with the `neighbors` matrix, this can make easy the design of some search algorithms (such as looking for all elements connected to a node).

Shape qualities

Vector `shape_qualities` (OUT) of size M (same size as the number of columns in the `connectM` matrix).

This vector gives the shape quality of each element.

The formula for the shape quality of a triangle writes:

$$Q_s = 4\sqrt{3}\,\frac{S}{L_{\max}\,P}$$

with:

S	Area of the triangle.
$L_{\max}$	Max length of the four sides and the two diagonals.
P	Perimeter of the triangle.

The formula for the shape quality of a 2-D quadrangle writes:

$$Q_s = 8\sqrt{2}\,\frac{S_{\min}}{L_{\max}\,P}$$

with:

$S_{\min}$	Minimum area of the four triangles.

[36] *A subdomain means a set of connected elements (not fully separated by a hard edge boundary).*
[37] *If the initial color vector is empty, all elements (initial and new ones) are affected with color 1*

$L_{\max}$ Max length of the four sides and the two diagonals.

P Perimeter of the quadrangle.

The formula for the shape quality of a 3-D quadrangle writes:

$$Q_s^{3D} = Q_s^{2D}\ Q_w$$

with:

Q_w Warp quality of the quadrangle:

$$Q_w = 1 - \frac{a\cos\left(\max\left(\langle N_0, N_2\rangle, \langle N_1, N_3\rangle\right)\right)}{\pi}$$

N_i Normal to the quad at node i.

Histograms

The `histo_Qs` and `histo_Qh` histograms can be used to check for the shape and size qualities of the mesh.
In the anisotropic case, the shape quality of an element is computed using the metrics at its nodes (minimum value of the qualities computed with the nodal metric transformations).
`histo_Qs` is the histogram of the `shape_qualities` vector.
`histo_Qh` is computed only when the option flag `compute_Qh_flag` is up (see § III-3).
Each histogram stores the minimum, the maximum and the average value as data members.

Errors and warnings

`error_code` and `warning_code are two enums` which give upon exit the error and warning codes if any. The string `msg1` holds explanations messages about the error/warning raised. In case of error, the meshing process is aborted and the output mesh (`connectM`) is empty. With a mere warning, the process goes to its end, though the final mesh may have a poor quality.
A correct run gives the `CM2_NO_ERROR` and `CM2_NO_WARNING` codes (zero value).
See § III-2 for more detailed explanations of the error and warning codes.

Complementary information

We gather in this section all the remaining fields of the data structure. They are all output values about the final mesh:

- The number of elements in the mesh - also equals to the number of columns in `connectM`.
- The number of quads (always null with TriaMesh & TriaMesh Aniso) and the number of triangles (null with QuadMesh & QuadMesh Aniso when `all_quad_flag = true`). The sum equals to the number of elements above.
- The number of nodes in the mesh (less or equal to the number of columns in `pos`).
- The number of input and output hard edges. In strict mode (see § III-3), these two quantities must be equal.
- The number of input and output hard nodes. In strict mode, these two quantities must be equal.
- The area of the mesh.
- The area of the quads (always null with TriaMesh & TriaMesh Aniso) and the area of the triangles (null with QuadMesh & QuadMesh Aniso when `all_quad_flag = true`). The sum equals to the area of the mesh above.
- The bounding box of the mesh
- The worst shape quality of the elements in the mesh upon entry (`REGULARIZE_MODE` only).
- The worst shape quality of the elements in the final mesh - also available in `histo_Qs`.
- The number of subdomains - i.e. the number of different values in the `colors` vector.
- The orientation of the external boundary: +1 if the contour is counter-clockwise, -1 if the contour is clockwise.
- The error and warning codes (see III-2).
- The error message.

- The times spent in the three successive steps of the mesher.
- The total time spent in the mesher.
- The global speed of the mesher (number of generated elements per second).

```
struct data_type
{
   DoubleMat               pos;
   UIntMat                 connectB;
   UIntVec                 isolated_nodes;
   UIntVec                 repulsive_points;
   UIntMat                 background_mesh;
   DoubleVec               metrics;
   UIntMat                 connectM;

   UIntVec                 unenforced_boundary_IDs;
   UIntVec                 unenforced_node_IDs;
   UIntVec                 pathological_boundary_IDs;
   UIntVec                 colors;
   UIntMat                 neighbors;
   UIntVec                 ancestors;
   DoubleVec               shape_qualities;
   misc::histogram         histo_Qs;
   misc::histogram         histo_Qh;

   unsigned                nefs;
   unsigned                nefs_Q4;
   unsigned                nefs_T3;
   unsigned                nods;
   unsigned                total_nods;
   unsigned                hard_nodes_in;
   unsigned                hard_edges_in;
   unsigned                hard_nodes_out;
   unsigned                hard_edges_out;
   double                  area;
   double                  area_Q4;
   double                  area_T3;
   DoubleVec2              min_box;
   DoubleVec2              max_box;
   double                  Qmin_in;
   double                  Qmin;
   unsigned                subdomains;
   int                     boundary_sgn;

   double                  front_time;
   double                  refine_time;
   double                  optim_time;
   double                  total_time;
   double                  speed;

   error_type              error_code;
   warning_type            warning_code;
   char                    msg1[256];
};
```

Table 2 – `triamesh::mesher::data_type` and `quadmesh::mesher::data_type` (only the data members are shown).

```
struct data_type
{
   DoubleMat               pos;
   UIntMat                 connectB;
   UIntVec                 isolated_nodes;
   UIntVec                 repulsive_points;
   UIntMat                 background_mesh;
   DoubleMat               metrics;
   UIntMat                 connectM;

   UIntVec                 unenforced_boundary_IDs;
   UIntVec                 unenforced_node_IDs;
   UIntVec                 pathological_boundary_IDs;
   UIntVec                 colors;
   UIntMat                 neighbors;
   UIntVec                 ancestors;
   DoubleVec               shape_qualities;
   misc::histogram         histo_Qs;
   misc::histogram         histo_Qh;

   unsigned                nefs;
   unsigned                nefs_Q4;
   unsigned                nefs_T3;
   unsigned                nods;
   unsigned                total_nods;
   unsigned                hard_nodes_in;
   unsigned                hard_edges_in;
   unsigned                hard_nodes_out;
   unsigned                hard_edges_out;
   double                  area;
   double                  area_Q4;
   double                  area_T3;
   DoubleVec2              min_box;
   DoubleVec2              max_box;
   double                  Qmin_in;
   double                  Qmin;
   unsigned                subdomains;
   int                     boundary_sgn;

   double                  front_time;
   double                  refine_time;
   double                  optim_time;
   double                  total_time;
   double                  speed;

   error_type              error_code;
   warning_type            warning_code;
   char                    msg1[256];
};
```

Table 3 – `triamesh_aniso::mesher::data_type` and `quadmesh_aniso::mesher::data_type` (only the data members are shown).

III-2 ERROR AND WARNING CODES

Error codes

```
enum error_type
{
   CM2_NO_ERROR,                    // 0
   CM2_LICENSE_ERROR,               // -100
   CM2_MODE_ERROR,                  // -101
   CM2_DATA_ERROR,                  // -102
   CM2_NODES_LIMIT_ERROR,           // -103
   CM2_NODE_ERROR,                  // -104
   CM2_EDGE_ERROR,                  // -105
   CM2_BOUNDARY_ERROR,              // -106
   CM2_DEGENERATED_ELEMENT,         // -107
   CM2_BACKGROUND_MESH_ERROR,       // -108
   CM2_SYSTEM_MEMORY_ERROR,         // -199
   CM2_INTERNAL_ERROR               // -200
};
```

Table 4 – Error codes for CM2 TriaMesh and CM2 TriaMesh Aniso.

```
enum error_type
{
   CM2_NO_ERROR,                    // 0
   CM2_LICENSE_ERROR,               // -100
   CM2_MODE_ERROR,                  // -101
   CM2_DATA_ERROR,                  // -102
   CM2_NODES_LIMIT_ERROR,           // -103
   CM2_NODE_ERROR,                  // -104
   CM2_EDGE_ERROR,                  // -105
   CM2_BOUNDARY_ERROR,              // -106
   CM2_BOUNDARY_PARITY_ERROR,       // -107
   CM2_IRREGULAR_BOUNDARY_ERROR,    // -108
   CM2_DEGENERATED_ELEMENT,         // -109
   CM2_BACKGROUND_MESH_ERROR,       // -110
   CM2_SYSTEM_MEMORY_ERROR,         // -199
   CM2_INTERNAL_ERROR               // -200
};
```

Table 5 – Error codes for CM2 QuadMesh and CM2 QuadMesh Aniso.

The error code is located in the data structure (`triamesh::mesher::data_type` and `quadmesh::mesher::data_type`).

Example:

```
if (data.error_code != triamesh::mesher::data_type::CM2_NO_ERROR)
{
  // Error, do something.
}
```

`CM2_NO_ERROR`	OK, no problem.
`CM2_LICENSE_ERROR`	The registration must occur before the instantiation of any meshers. Check also your license. You may renew it. Please, contact license@computing-objects.com
`CM2_MODE_ERROR`	The operating mode is not valid (see § III-3). Check the positivity/range of scalar values such as `shape_quality_weight`, `max_gradation`...
`CM2_DATA_ERROR`	The input data are not valid. Check the sizes of matrices, of vectors, node indices in the connectivity matrices, look for insane values...
`CM2_NODES_LIMIT_ ERROR`	The limitation on nodes number is too low.
`CM2_NODE_ERROR`	Invalid hard node(s): some hard nodes are considered coincident or a node (`isolated_nodes`) is located outside the domain. Strict mode only.
`CM2_EDGE_ERROR`	Invalid hard edge(s): at least one hard edge is intersecting another hard entity (node, edge) or is located outside the domain. Strict mode only.
`CM2_BOUNDARY_ERROR`	The external boundary mesh is not closed. The algorithm cannot distinguish the interior from the exterior.
`CM2_BOUNDARY_ PARITY_ERROR`	At least one of the hard lines (external boundary or internal line) has an odd number of edges. QuadMesh only.
`CM2_IRREGULAR_ BOUNDARY_ERROR`	At least one of the hard lines (external boundary or internal line) is too irregular. In case of a polygonal line, try to enforce the even condition on each segment of the line. QuadMesh only.
`CM2_DEGENERATED_ ELEMENT`	At least one of the elements is invalid (null or negative surface). Check the hard edges and hard nodes.
`CM2_BACKGROUND_MESH_ ERROR`	The background mesh is not valid (not a triangle mesh, nodes not in the pos matrix or degenerated elements).
`CM2_SYSTEM_MEMORY_ ERROR`	Insufficient memory available. Mesh is too big to be generated (over several tens millions elements).
`CM2_INTERNAL_ERROR`	Unknown cause of error. Contact support.

Table 6 – Error codes.

This table reflects the priority of the error codes. The highest in the table, the highest priority.

For error codes `CM2_DEGENERATED_ELEMENT` and `CM2_INTERNAL_ERROR`, save the data by calling `data_type::save` and send the zipped file to support@computing-objects.com.

Warning codes

```
enum warning_type
{
   CM2_NO_WARNING,                    // 0
   CM2_INTERRUPTION,                  // -10
   CM2_NODES_LIMIT_WARNING,           // -11
   CM2_NODE_DISCARDED,                // -12
   CM2_EDGE_DISCARDED,                // -13
   CM2_SHAPE_QUALITY_WARNING          // -14
};
```

Table 7 – Warning codes for all meshers.

The warning code is located in the data structure `data_type`.

Example:

```
if (data.warning_code ==
    triamesh::mesher::data_type::CM2_SHAPE_QUALITY_WARNING)
{
  // Warning, do something.
}
```

`CM2_NO_WARNING`	OK, no problem.
`CM2_INTERRUPTION`	The run has been aborted by the user (through the interrupt handler). The final mesh may be empty or valid but of poor quality.
`CM2_NODES_LIMIT_WARNING`	The node limit has been reached and the mesh may be far from optimal.
`CM2_NODE_DISCARDED`	Invalid hard node(s): some hard nodes are considered coincident (only one node is kept, the lowest one) or a node (`isolated_nodes`) is located outside the domain. Non strict mode only.
`CM2_EDGE_DISCARDED`	Invalid hard edge(s): at least one hard edge is intersecting another hard entity (node, edge) or is located outside the domain. Non strict mode only.
`CM2_SHAPE_QUALITY_WARNING`	The final mesh is valid but at least one of the elements is very bad (shape quality < 0.01). Check that the discretizations of connected lines are not too different.

Table 8 – Warning codes.

This table reflects the priority of the warning codes. The highest in the table, the highest priority.

For warning code `CM2_SHAPE_QUALITY_WARNING`, if the boundary mesh is good, save the data by calling `data_type::save` and send the zipped file to support@computing-objects.com.

III-3 OPTIONS OF THE MESH GENERATORS

CM2 TriaMesh and CM2 QuadMesh and their anisotropic counterparts can be used as regular meshers or as optimizers of some already existing meshes. For that matter, flags and parameters can be used to adapt the meshers to many various needs. Some options are relevant only in the meshing mode, some only in the optimizing mode. They are all gathered into a structure of type `operating_mode_type` as a public member field of the meshers:

```
cm2::triamesh::mesher::operating_mode_type          mode;
cm2::quadmesh::mesher::operating_mode_type          mode;
cm2::triamesh_aniso::mesher::operating_mode_type    mode;
cm2::quadmesh_aniso::mesher::operating_mode_type    mode;
```

From this point and all along this section, we will make no distinction between the isotropic meshers and their anisotropic counter-parts, as they have the very same options. For instance, a mention to CM2 QuadMesh will stand for both CM2 QuadMesh and CM2 QuadMesh Aniso.

Basic operating mode

`basic_mode`. Default = `MESH_MODE`.
The meshers have three distinct operating modes:

- `MESH_MODE`. This is the default mode. A 2-D mesh is generated from a set of hard edges and possibly some internal hard nodes. The `connectB` matrix and `isolated_nodes` in the data structure are input fields whereas the `connectM` matrix is an output of the mesher.
- `REGULARIZE_MODE`. The mesher is only used to improve the quality of an already existing mesh. The `connectM` matrix is both input and output of the mesher.
- `CONVEX_HULL_MODE`. The mesher builds a triangulation of a set of points. This is the approximate convex hull of the points. No boundary (`connectB`) is needed here, only the `isolated_nodes` are used. No new nodes are added to the mesh. Note that the convex hull is always a triangle mesh, even with QuadMesh.

Strict constraints enforcement

`strict_constraints_flag`. Default = `true`. Used in `MESH_MODE` only.

This flag tells the mesher to stop on an aborting error (`CM2_EDGE_ERROR`, `CM2_NODE_ERROR`) if at least one of the constraints cannot be enforced or to simply issue a warning (`CM2_EDGE_DISCARDED`, `CM2_NODE_DISCARDED`) and go on with the process if possible.
Beware that a rejected edge can make a contour mesh unclosed and then lead to a `CM2_BOUNDARY_ERROR`.

Keeping or removing internal holes

`subdomains_forcing`. Default = `0`. Used in `MESH_MODE` only.

Forcing mode for intern subdomains. Possible values are:

- -1: All intern subdomains are considered as holes regardless of the orientation of their enclosing boundary.
- 0: Intern subdomains are considered as holes when the orientation of their enclosing boundary is opposite to the orientation of the most enclosing domain. Otherwise, they are meshed.
- +1: All intern subdomains are meshed regardless of the orientation of their enclosing boundary.

	strict_constraints_flag	
	true	**false**
Two hard nodes are coincident (or too close from each other)	Error CM2_NODE_ERROR	Warning CM2_NODE_DISCARDED The highest node is rejected.
A hard node is located inside a hard edge.	Error CM2_EDGE_ERROR	Warning CM2_EDGE_DISCARDED The edge is rejected.
Two hard edges cross each other.	Error CM2_EDGE_ERROR	Warning CM2_EDGE_DISCARDED The highest edge is rejected.
A hard node is located outside the domain (or in a hole).	Error CM2_NODE_ERROR	Warning CM2_NODE_DISCARDED The node is rejected.
A hard edge is located outside the domain (or in a hole) but with nodes on boundary.	Error CM2_EDGE_ERROR	Warning CM2_EDGE_DISCARDED The edge is rejected..
A hard edge is located outside the domain (or in a hole), with nodes ouside also.	Error CM2_NODE_ERROR	Warning CM2_NODE_DISCARDED The node(s) and the edge are rejected.
The contour mesh is not close.	Error CM2_BOUNDARY_ERROR	Error CM2_BOUNDARY_ERROR

Table 9 – Effects of the `strict_constraints_flag` on invalid constraints.

All-quad or quad-dominant mode (CM2 QuadMesh & CM2 QuadMesh Aniso)

`all_quad_flag`. Default = `true`. Used in `MESH_MODE` only.

This flag tells the mesher to generate only quadrangles. When this flag is on, parity of the 1-D meshes along the boundaries and internal lines is generally needed and sufficient to get an all-quad mesh. Otherwise the mesher may fail with a `CM2_BOUNDARY_PARITY` error[38].
Note that when this flag is off (quad-dominant mode) triangles are not necessarily present in the final mesh (i.e. an all-quad mesh may still be achieved in some cases).
Usually the mesher generates better meshes in quad-dominant mode because of this possibility to use triangles.

Refinement

`refine_flag`. Default = `true`. Used in `MESH_MODE` only.

This flag enables the generation of new points inside the domain in order to get good element / size qualities. For TriaMesh, this flag off makes the mesher to triangulate only the domain, i.e. it stops after the front mesh step. Only the hard nodes will be in the final mesh and matrix `pos` will be unchanged. For QuadMesh however, a minimal amount of new nodes has to be generated in order to mesh a domain with quads only, even with `refine_flag = false`.

[38] *The rule of thumb for all-quad meshing is that the domain defined by the twice-coarser boundary (using every other boundary node) is valid (no self intersection).*

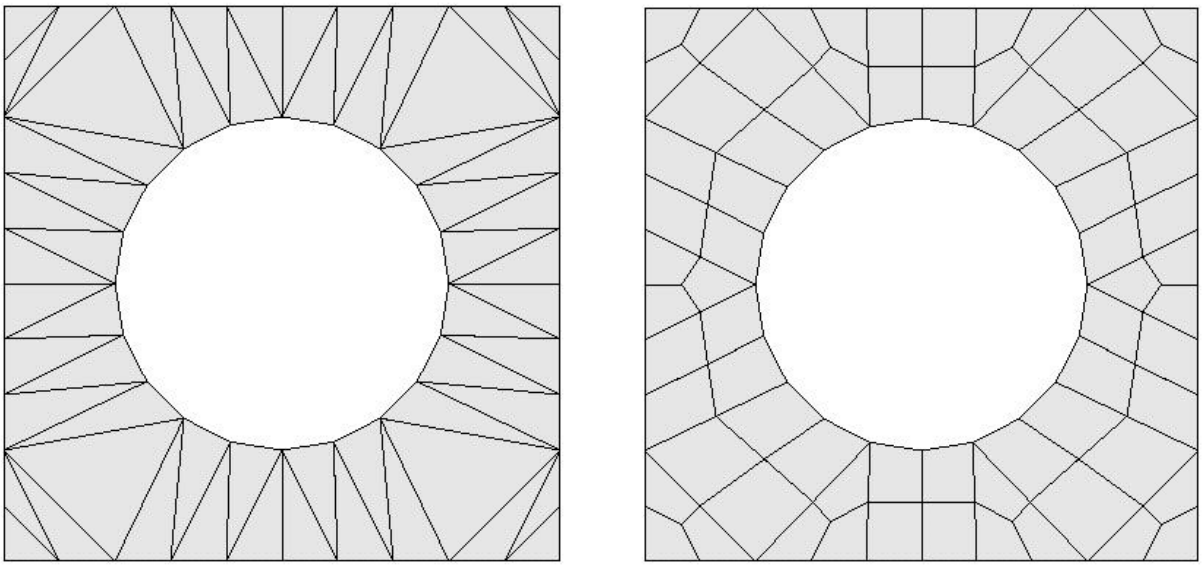


Figure 34 – Mode "`refine_flag = false`" (T3 and Q4) for the example II-3.

Recomputing the boundary edges

`reset_external_boundaries_flag`. Default = `false`.
Used in `REGULARIZE_MODE` and `CONVEX_HULL_MODE`.

This flag is useful when the user wants to regularize a 2-D mesh without having its boundaries[39], or to get the boundary of the convex hull of a set of points. When this flag is up, the mesher will search for all edges that are connected to only one element. These edges will be considered as hard edges and appended to the input `connectB` matrix (if not already there). The edges present upon entry in `connectB` will remain in the matrix and will also be considered as hard edges by the optimizer.
Internal hard nodes cannot be automatically detected. They must be specified by the user in the `isolated_nodes` vector so that they are not smoothed or eliminated.

Node smoothing

`node_smoothing_flag`. Default = `true`. Used in `MESH_MODE` and `REGULARIZE_MODE`.

This flag controls the node-smoothing scheme in the optimization step.
Node smoothing doesn't change the mesh connectivity, only the coordinates of nodes.
This flag has no effect when the optimization step is skipped (`optim_level` = 0)

Node inserting

`node_inserting_flag`. Default = `true`. Used in `MESH_MODE` and `REGULARIZE_MODE`.

This flag controls the node-inserting scheme in the optimization step.
Node inserting increases the number of nodes, changes the mesh connectivity, but doesn't change the other nodes' coordinates.
This flag has no effect when the optimization step is skipped (`optim_level` = 0)

Node removing

`node_removing_flag`. Default = `true`. Used in `MESH_MODE` and `REGULARIZE_MODE`.

[39] *Set `reset_external_boundaries_flag = true` and `optim_level = 0` (cf. below) to only extract the boundary of a 2D mesh without any change on it.*

This flag controls the node-removing scheme in the optimization step.
Node removing decreases the number of nodes, changes the mesh connectivity, but doesn't change the other nodes' coordinates.
This flag has no effect when the optimization step is skipped (`optim_level` = 0)

Shell remeshing

`shell_remeshing_flag`. Default = `true`. Used in `MESH_MODE` and `REGULARIZE_MODE`.

This flag controls the edge-swapping scheme in the optimization step.
Edge swapping changes the mesh connectivity, but doesn't change the number of nodes nor their coordinates.
This flag has no effect when the optimization step is skipped (`optim_level` = 0)

Avoiding clamped edges (CM2 TriaMesh & CM2 TriaMesh Aniso)

`no_clamped_edges_flag`. Default = false. Used in `MESH_MODE` and `REGULARIZE_MODE`.

This is a special node-inserting scheme dedicated to breaking the non-hard edges with two hard nodes.
This option supersedes `node_inserting_flag`. It increases the number of nodes, changes the mesh connectivity, but doesn't change the other nodes' coordinates.
This flag has no effect when the optimization step is skipped (`optim_level` = 0)

Computation of the size-qualities histogram

`compute_Qh_flag`. Default = `false`. Used in all modes.

Before exiting the process, this flag tells the mesher to compute the histogram of the size quality of all the edges in the mesh. The users are rarely interested in this histogram, so the default state is `false`.

Pattern for structured meshes (CM2 TriaMesh & CM2 TriaMesh Aniso)

`structured_pattern`. Default = -1. Used in `MESH_MODE` only.

This option controls the way the generators does the structured meshes when possible (on rectangular-like domains).
In TriaMesh & TriaMesh Aniso, `structured_pattern` can take four values:

- -1: This is the default mode. The triangular meshes are always done with the frontal-Delaunay algorithm. This usually gives “optimal” meshes.
- 0: When possible, generates structured left-oriented meshes (simply oriented pattern).
- +1: When possible, generates structured right-oriented meshes (simply oriented pattern).
- +2: When possible, generates structured UJ meshes (“Union Jack” pattern).

Pattern for structured meshes (CM2 QuadMesh & CM2 QuadMesh Aniso)

`structured_flag`. Default = `true`. Used in `MESH_MODE` only.

This option controls the way the generators do the structured meshes when possible (on rectangular-like domains):

- `true`: This is the default mode. When possible, generates structured (grid-like) meshes.

- `false`: The quad meshes are always done with the frontal-Delaunay algorithm[40].

Multi-structured subdomains

`multi_structured_flag`. Default = `false`. Used in `MESH_MODE` only.

Triggers a more complete search for structurable subdomains than normally (at the price of a lower speed). With this flag on, several rectangular subdomains can be meshed structurably, whereas it is not always the case with this flag down.

Used only when `refine_flag = true` and `structured_flag = true` (QuadMesh), `structured_pattern` != -1 (TriaMesh) and when there is no background mesh, no isolated nodes and null `max_gradation` or null `target_metric`.

Limit on the number of nodes

`nodes_limit`. Default = `UINT_MAX`. Used in all modes.

When this limit is reached by the mesher[41], the `CM2_NODES_LIMIT_WARNING` is issued. The algorithm generates a mesh but the quality can be far from optimal. When the limit is so low that the mesher cannot even insert all the hard nodes, the `CM2_NODES_LIMIT_ERROR` is raised and the mesh is aborted.

Optimization level

`optim_level`. Integer between 0 and 10. Default = 3. Used in `MESH_MODE` and `REGULARIZE_MODE`.

A null value makes the mesher to skip the optimization step. The speed is maximal but the quality may be poor. From value 1 on, the optimizer algorithm uses several techniques to improve both the shape quality and the size quality of the elements, such as node smoothing, edge swapping, node insertion and node removal. Level 3 is usually a good trade-off between quality and speed.

Target metric

`target_metric`. Double value (isotropic) or `DoubleSym2` (anisotropic).
Default = 0. Used in `MESH_MODE` only.

Element size inside the domain. The elements size tends toward this value as they get away from the hard (boundary) edges.
This parameter is often used to reduce the total number of elements (when target_metric > default sizes based on boundary edge lengths) and to save FEM computation time without loosing much on element shape quality.
The target_metric may be smaller or bigger than the default sizes based on boundary edge lengths.

Max gradation

`max_gradation`. Double value greater than 0. Default = 0.5. Used in `MESH_MODE` only.

This parameter controls the gradation of the elements size from the boundary sizes to the size defined by `target_metric` inside the domain.
A value close to 0 leads to a more progressive variation of mesh size (smoother).

---

40 This may give also structured meshes. The true option enforces this and is much faster whenever a structured mesh can be generated. Since release 3.4 this flag is less useful because QuadMesh is able to generate naturally a perfect mesh on rectangle domains or on domains such as shown in Figure 17, even with this flag off.
41 This limit is not strict. The number of nodes actually in the final mesh may be slightly different from this limit (lesser or greater).

Weight on shape quality

`shape_quality_weight`. Double value between 0 and 1. Default = 0.60
Used in `MESH_MODE` and `REGULARIZE_MODE`.

This parameter controls the trade-off between shape optimization and size optimization. It is the weight of the shape quality in the measure of the global quality of an element. The default value (0.6) gives a slight preference to the shape quality over the size quality.

Weight on quadrangles (CM2 QuadMesh & CM2 QuadMesh Aniso)

`quadrangle_weight`. Double value between 0 and 1. Default = 0.70
Used in `MESH_MODE` and `REGULARIZE_MODE`.

This parameter controls the trade-off between a higher ratio of quads and a better mesh with more triangles.

- With `quadrangle_weight` = 0, quadrangles are never used.
- With `quadrangle_weight` = 0.5, quadrangles are used only when they improve the quality of the mesh (when a quad is better than two triangles).
- For values between 0.5 and 1, quadrangles are more and more used even if this lead to a lesser global quality of the mesh.
- With `quadrangle_weight` = 1, the minimum number of triangles are used to get a valid mesh (but may be of poor quality).

The default value (0.70) gives a significant preference to the quad/triangle ratio over the mesh quality.

Minimum quad quality (CM2 QuadMesh & CM2 QuadMesh Aniso)

`min_Q4_angle_quality`. Double value between 0 and 1. Default = 0 (no minimum).
Used in `MESH_MODE`.

Minimum acceptable angle quality for the quadrangles.
This parameter is taken into account in mixed mode only (`all_quad_flag = false`).

This quality threshold is based on the *angle* quality of the quads (not the *geometric* quality which takes the length ratios also into account). The angle quality is computed as the minimum of the four angles at sommits[42]. Set min_Q4_angle_quality = 1 to allow rectangles only (quads with right angles only).
In this case, be aware that when boundaries are not straight very few rectangles may be generated (mostly triangles).

Upper bound on edges length

`length_upper_bound`. Double value greater than 0. Default = 1.414. Used in `MESH_MODE` only.

This parameter is used to limit the length of the edges in the generated mesh (normalized length). This is *not* a strict enforcement however. Most of the edges will be shorter than this limit, but some may remain somewhat longer. The default value (1.414) gives the optimal meshes with respect to the size qualities. With this default value, the average edge length tends to be 1 (optimal edge quality on average).
Sometimes, it can be useful to limit the length of the edges to a shorter value (usually between 1 and 1.414), and to accept an average value smaller than 1 (sub-optimal edge qualities on average).

Display handler

`display_hdl`. Default = `NULL`. Used in all modes.

[42] The angle quality of a rectangle equals to 1 (perfect) whereas the geometric quality is equal to 1 only for a square.

This user-supplied function is used to handle the messages issued by the mesher.

```
typedef void (*display_handler_type) (void* pass_thu,
                                      unsigned level, const char* msg);
```

The `pass_thru` parameter is the pointer set by the user in the operating mode structure.
The level parameter gives the importance of the message:

- +0 ➔ important (for instance entering a major step of the process)
- +1 ➔ somewhat important (minor step of the process)
- ≥ 2 ➔ not serious (debug messages that should not be printed for end-users).

The `msg` parameter is the string message (length <= 255 characters).

Note:
This handler is not called in case of error or warning. At the end of the run, the user must check for an error or a warning in the fields `data_type::error_code` and `data_type::warning_code` and then (in case of error or warning) process the string `data_type::msg1`.

Example:

```
void my_display_hdl (void* pass_thru, unsigned level, const char* msg)
{
   window_type*   my_window = static_cast<window_type*> (pass_thru);
   my_window->show (msg);
}

cm2::triamesh::mesher             my_mesher;
cm2::triamesh::mesher::data_type  my_data (pos, connectB);
window_type                       my_window;   // A “window” instance.

my_mesher.mode.display_handler = &my_display_handler;
my_mesher.mode.pass_thru = static_cast<void*> (&my_window);
my_mesher.run (my_data);      // Will call my_display_hdl with “my_window”
                              // in pass_thru parameter.
```

Interrupt handler

`interrupt_hdl`. Default = `NULL`. Used in all modes.

Can be useful for big meshes (over hundreds of thousands of elements).

```
typedef bool (*interrupt_handler_type)( void* pass_thu, double progress);
```

This handler, if any, is called periodically by the mesher to check for a stop signal. When the handler returns `true`, the mesher aborts its current step. If the interruption occurs early in the meshing stage - for instance in the front mesh step - the mesh is invalid, so it is cleared. From the refine step on, however, the user can get a valid mesh upon exit, though probably of poor quality.
An interruption also raises the `CM2_INTERRUPTION` warning.

The `pass_thru` parameter is the pointer set by the user in the operating mode structure (the same parameter is also passed to the display handler).
The parameter `progress` (between 0 and 1) gives a hint about the progress of the meshing.

Example:

```
bool my_interrupt_handler (void* pass_thu, double progress)
{
   clock_t*   t_limit = static_cast<clock_t*> (pass_thru);
   return clock() > (*t_limit);
}

cm2::triamesh::mesher              my_mesher;
cm2::triamesh::mesher::data_type   my_data (pos, connectB);
clock_t                            my_limit (clock() + 1E3*CLOCKS_PER_SEC);

my_mesher.mode.interrupt_handler = &my_interrupt_handler;
my_mesher.mode.pass_thru = static_cast<clock_t*> (&my_limit);
my_mesher.run (my_data);         // Will stop if duration > 1000 s.
```

```
enum basic_mode_type
{
   MESH_MODE,
   REGULARIZE_MODE,
   CONVEX_HULL_MODE
};


struct operating_mode_type
{
   basic_mode_type         basic_mode;
   bool                    strict_constraints_flag;
   int                     subdomains_forcing;
   bool                    refine_flag;
   bool                    reset_external_boundaries_flag;
   bool                    node_smoothing_flag;
   bool                    node_inserting_flag;
   bool                    node_removing_flag;
   bool                    shell_remeshing_flag;
   bool                    no_clamped_edges_flag;
   bool                    compute_Qh_flag;
   int                     structured_pattern;
   bool                    multi_structured_flag;
   unsigned                nodes_limit;
   unsigned                optim_level;
   double                  target_metric;
   double                  max_gradation;
   double                  shape_quality_weight;
   double                  length_upper_bound;
   display_handler_type    display_hdl;
   interrupt_handler_type  interrupt_hdl;
   void*                   pass_thru;
};
```

Table 10 – `triamesh::operating_mode_type`
(only the data members are shown).

```
enum basic_mode_type
{
   MESH_MODE,
   REGULARIZE_MODE,
   CONVEX_HULL_MODE
};


struct operating_mode_type
{
   basic_mode_type         basic_mode;
   bool                    strict_constraints_flag;
   int                     subdomains_forcing;
   bool                    refine_flag;
   bool                    reset_external_boundaries_flag;
   bool                    node_smoothing_flag;
   bool                    node_inserting_flag;
   bool                    node_removing_flag;
   bool                    shell_remeshing_flag;
   bool                    no_clamped_edges_flag;
   bool                    compute_Qh_flag;
   int                     structured_pattern;
   bool                    multi_structured_flag;
   unsigned                nodes_limit;
   unsigned                optim_level;
   DoubleSym2              target_metric;
   double                  max_gradation;
   double                  shape_quality_weight;
   double                  length_upper_bound;
   display_handler_type    display_hdl;
   interrupt_handler_type  interrupt_hdl;
   void*                   pass_thru;
};
```

Table 11 – `triamesh_aniso::operating_mode_type`
(only the data members are shown).

```
enum basic_mode_type
{
   MESH_MODE,
   REGULARIZE_MODE,
   CONVEX_HULL_MODE
};


struct operating_mode_type
{
   basic_mode_type         basic_mode;
   bool                    strict_constraints_flag;
   int                     subdomains_forcing;
   bool                    all_quad_flag;
   bool                    refine_flag;
   bool                    reset_external_boundaries_flag;
   bool                    node_smoothing_flag;
   bool                    node_inserting_flag;
   bool                    node_removing_flag;
   bool                    shell_remeshing_flag;
   bool                    compute_Qh_flag;
   bool                    structured_flag;
   bool                    multi_structured_flag;
   unsigned                nodes_limit;
   unsigned                optim_level;
   double                  target_metric;
   double                  max_gradation;
   double                  shape_quality_weight;
   double                  quadrangle_weight;
   double                  min_Q4_angle_quality;
   double                  length_upper_bound;
   display_handler_type    display_hdl;
   interrupt_handler_type  interrupt_hdl;
   void*                   pass_thru;
};
```

Table 12 – `quadmesh::operating_mode_type`
(only the data members are shown).

```
enum basic_mode_type
{
   MESH_MODE,
   REGULARIZE_MODE,
   CONVEX_HULL_MODE
};


struct operating_mode_type
{
   basic_mode_type         basic_mode;
   bool                    strict_constraints_flag;
   int                     subdomains_forcing;
   bool                    all_quad_flag;
   bool                    refine_flag;
   bool                    reset_external_boundaries_flag;
   bool                    node_smoothing_flag;
   bool                    node_inserting_flag;
   bool                    node_removing_flag;
   bool                    shell_remeshing_flag;
   bool                    compute_Qh_flag;
   bool                    structured_flag;
   bool                    multi_structured_flag;
   unsigned                nodes_limit;
   unsigned                optim_level;
   DoubleSym2              target_metric;
   double                  max_gradation;
   double                  shape_quality_weight;
   double                  quadrangle_weight;
   double                  min_Q4_angle_quality;
   double                  length_upper_bound;
   display_handler_type    display_hdl;
   interrupt_handler_type  interrupt_hdl;
   void*                   pass_thru;
};
```

Table 13 – `quadmesh_aniso::operating_mode_type` (only the data members are shown).

III-4 GENERAL SCHEME OF THE GENERATORS

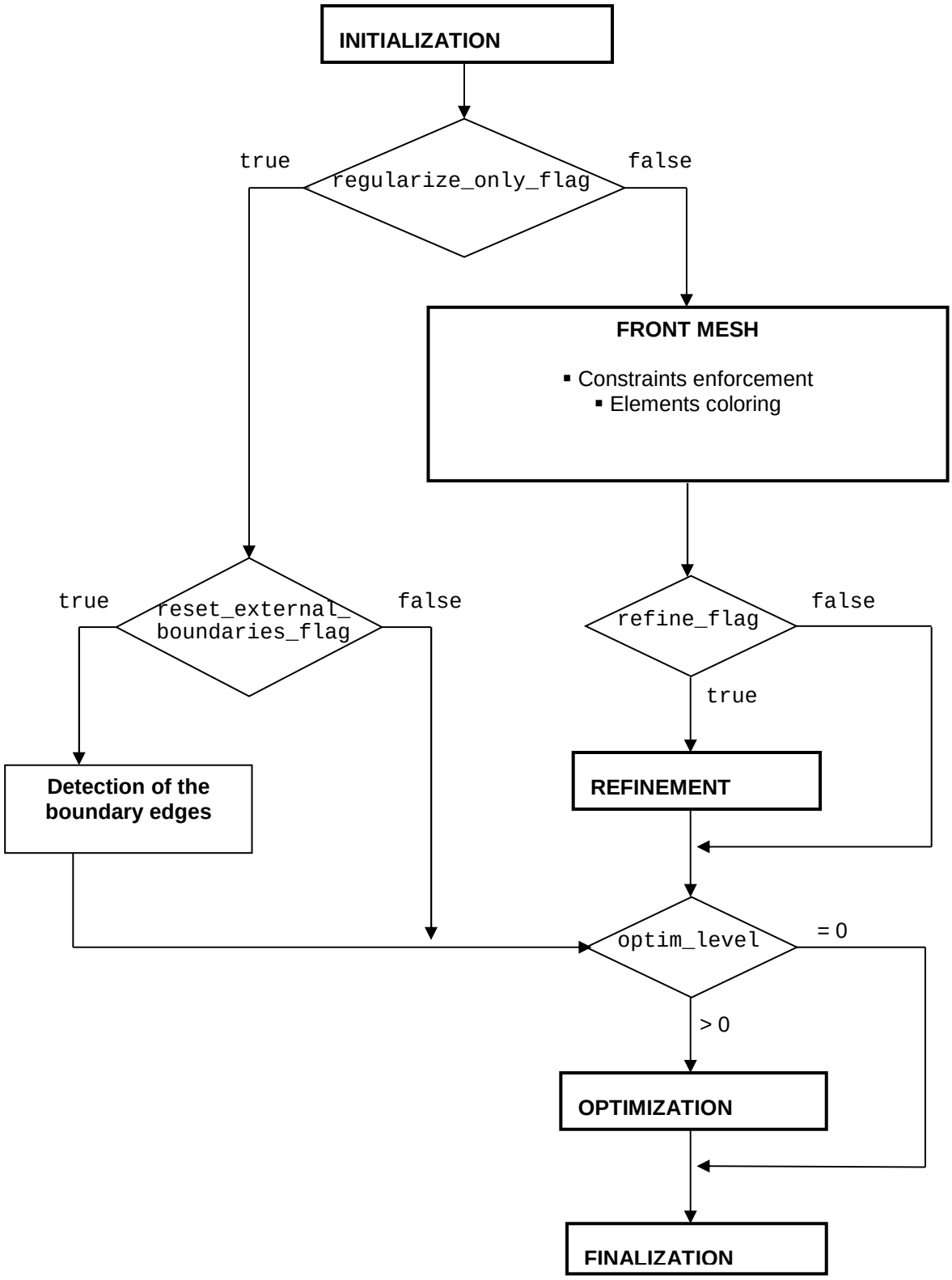


Figure 35 – General scheme of the mesh generators.

